# moles 

USER MANUAL

Version 4.6

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## 1. Getting started

### 1.1. Introduction

### 1.1.1. General

MOVE3 is a software package developed by Sweco Nederland for the Design and Adjustment of 3D, 2D and 1D geodetic networks. MOVE3 fully complies with the requirements and specifications of the Delft theory of network design and adjustment. This theory is generally acknowledged as the most efficient tool for processing and Quality Control of survey data.

MOVE3 properly handles all complex mathematics associated with 3D networks. Thus, 3D adjustments are carried out in a true 3D mathematical model, without simplifications or compromises. In addition to 3D adjustments, the software can perform 2D and 1D adjustments as well. This feature is controlled by the so-called Dimension Switch.

MOVE3 can manage all geodetic observation types occurring in virtually any combination. When the observations allow a 3D solution, MOVE3 solves position and height. Likewise, a 2D or 1D solution can be obtained, provided that the available observations suffice for such a solution. Principally, this is the only requirement for MOVE3 to process networks.

MOVE3 is easy to operate, requiring a minimum of training. The Windows graphical user interface (GUI) utilises pull down menus, intelligent graphical linked data editors, mouse control, standard ASCII file I/O and advanced on-line help.

### 1.1.2. About this Manual

The MOVE3 User Manual is divided into the following parts:

- Chapter 1: Getting started, contains general information about the software, including a description of the installation procedure.
- Chapter 2: Using MOVE3, provides condensed information about the use of the MOVE3 software. More comprehensive information can be found in the on-line help utility of the MOVE3 Windows user interface.
- Chapter 3: Geodetic Concepts, presents a synopsis of some of the theoretical elements of MOVE3: reference systems, map projections, GNSS/GPS and detail measurements.
- Chapter 4: Quality Control, is devoted to the tools for Quality control implemented in MOVE3. A major portion of PART IV deals with the Delft theory of network design and adjustment.
- Chapter 5: Lists, contains additional, software related, technical information. This part also includes a literature list, a glossary and an index.

The user does not have to read the manual from beginning to end before getting started. Following the installation of the software, one may start immediately with the tutorial. The use of the software is described in chapter 2, which contains references to subjects further explained in chapter 3 and 4 . Thus the user may refer to the latter parts of the manual, when an additional explanation is required.

### 1.1.3. MOVE3 Specifications

Minimal system requirements:

- Windows 7/8/8.1/10;
- USB port.


## Capacity

The maximum network size is limited only by the available hardware.

## Observations

MOVE3 can handle both terrestrial and GNSS/GPS observations:

- Directions (up to 200 independent series per station);
- Distances (up to 10 scale factors per network);
- Zenith angles (up to 10 refraction coefficients per network);
- Azimuths (up to 10 azimuth offsets per network);
- Height differences;
- Local coordinates;
- Shift Vector;
- GNSS/GPS Baselines;
- Observed GNSS/GPS coordinates;
- Geometrical relations:
- angle between 3 points (incl. perpendicular);
- parallel lines (incl. mutual distance);
- co linearity;
- perpendicular lines;
- distance from point to line;
- chainage and offset;
- double distance;
- tape distance;
- identical.


## Processing Modes

3D, 2D and 1D geodetic networks can be processed in:

- Design mode, free network;
- Design mode, constrained network;
- Adjustment mode, free network;
- Adjustment mode, constrained network.


## Projections

The following projections are supported:
General:

- Transverse Mercator;
- Lambert;
- Stereographic;

Specific:

- RD (The Netherlands);
- Lambert 72 (Belgium);
- Gauss Krüger (Germany);
- Local (Stereographic);
- BRSO;
- Malaysian RSO;
- Swiss.


## Tools

MOVE3 includes separate tools such as fully automatic computation of approximate coordinates (COGO3), automatic loop detection and misclosure testing (LOOPS3) and adjustment pre-analysis (PRERUN3).

## General

- Open file specifications (ASCII files);
- Language support : English, Dutch;
- Interfaces with GNSS/GPS baseline processing packages (Leica, Sokkia, Topcon and Trimble);
- Interfaces with total station files from Leica, Topcon, Sokkia and Trimble;
- Interfaces with digital levelling files from Leica, Topcon, Sokkia and Trimble digital levels;
- Supported Geoid Model:
- Australia (AUSGeoid98 AUSGeoid09 and AUDGEOID2020)
- Belgium (hBG03 and hBG18)
- Canada (GPS*H, CGG2013a)
- Finland (FIN2000 and FIN2005)
- Latvia (LV98)
- New Zealand (NZGEOID2009)
- Portugal (PT08)
- Singapore (SGEOID09)
- Spain (EGM08-REDNAP)
- The Netherlands (NLGEO08 and NLGEO04)
- USA (GEOID99, GEOID03, GEOID09, GEOID12 and GEOID18)
- EGM96 and EGM2008
- Exporting DXF files;
- On-line help facilities.


### 1.2. Installation

### 1.2.1. Package Contents

The MOVE3 package contains the following components:

- MOVE3 installation package;
- MOVE3 USB hardware lock.


### 1.2.2. Hardware and Software Requirements

The following hardware is required to run MOVE3:

- PC with USB port;
- USB hardware lock (included in the MOVE3 package).

The following software is required to run MOVE3:

- Microsoft Windows 7/8/8.1/10;
- MOVE3 software;
- Sentinel system drivers or CodemeterRuntime.


### 1.2.3. Installation Procedure

Consult the installation guide for the installation procedure.

### 1.2.4. Starting MOVE3

To start MOVE3 click the MOVE3 item under Programs in the Windows Start menu. To start MOVE3 for Windows and immediately load a project use Explorer and double click the PRJ file.

### 1.2.5. Tutorial

See the MOVE3 tutorial to get familiar with processing networks in MOVE3.

## 2. Using MOVE3

### 2.1. Introduction

Part 2 of this manual provides the user with information about the use of the MOVE3 network adjustment software. The software consists of a Windows user interface and a number of computation modules. The Windows user interface provides the user with full control over all options and parameters necessary for the computation modules.

The options and parameters are described in the on-line help utility of the MOVE3 Windows user interface. The option parameters are examined in terms of defaults, possible values, and their effect on the software.

In this part of the manual the MOVE3 model is discussed. A major part is dedicated to one of the main features of MOVE3: the Dimension Switch. The use of the Dimension Switch itself is rather straightforward. The choice of dimension however, will have an effect on the handling of observation types. Therefore the chapter on the Dimension Switch is essential reading matter.

### 2.1.1. System Overview

All tasks, which are performed by MOVE3, are initiated from the Windows user interface. Hence the user never has to leave the user interface during an adjustment session in order to e.g. view or change the data. In general the data is managed, i.e. read, edited, displayed and saved, by the user interface and processed, i.e. prepared, checked and adjusted, by the other computation modules.

The software system comprises the following computation modules:

- COGO3 to compute approximate coordinates;
- GEOID3 to extract geoid heights from the support and installed geoid models;
- LOOPS3 to detect network loops and compute loop misclosures;
- PRERUN3 to perform pre-adjustment analysis;
- MOVE3 to perform design and adjustment computations.


### 2.2. MOVE3 Model

### 2.2.1. General

The mathematical model applied in MOVE3 is rigorously ellipsoidal, independent of the dimension of the solution. In essence an ellipsoid is a curved 2D surface. The third dimension is introduced by defining heights above the ellipsoid. The coordinate unknowns are therefore latitude, longitude and height. These unknowns are used internally. The user may supply approximate and known coordinates in a map projection (X East, Y North). In that case the adjusted coordinates will be output in the same projection, thus conveniently hiding the ellipsoidal model from the user (figure 2.2.1). The height used internally is an ellipsoidal height.

figure 2.2.1: Simplified structure of MOVE3.
When a geoid model is available (see paragraph 3.2.3, Geoid and Height Definition), the user can enter both orthometric heights and geoid heights. MOVE3 will transform the orthometric heights into ellipsoidal heights before the adjustment. After the adjustment the computed heights are re-transformed to and presented as orthometric heights. If a geoid model is not available the entered heights will be interpreted as ellipsoidal heights.

The mathematical model embodies a direct relation between the original input observations and the ellipsoidal unknowns. (Furtheron in this paragraph the importance of handling original observations will be explained.) The general form of the linearised model is expressed in
figure 2.2.2. The vector on the left contains the observations: directions (R), distances (S), zenith angles (Z), azimuths (A), height differences (DH), GNSS/GPS baselines elements (DX), observed GNSS/GPS coordinates (X) and geometrical relations (GR). The vector on the right, next to the design matrix $A$, contains the unknowns: latitude $(\varphi)$, longitude $(\lambda)$, height ( h ) and nuisance parameters ( np ).

## observations

$\left[\begin{array}{l}R_{i j} \\ S_{i j} \\ Z_{i j} \\ A_{i j} \\ D H_{i j} \\ D X_{i j} \\ X_{i j} \\ G R_{i j k} \\ G R_{i j k l}\end{array}\right]=\left[\begin{array}{lll}\cdots & \cdots & \cdots \\ \cdots & A & \cdots \\ \cdots & \cdots & \cdots\end{array}\right]$
unknowns
$\left[\begin{array}{l}\varphi_{i} \\ \lambda_{i} \\ h_{i} \\ \varphi_{j} \\ \lambda_{j} \\ h_{j} \\ : \\ n p \\ :\end{array}\right]$
figure 2.2.2: MOVE3 mathematical model
In the adjustment the unknowns are not computed at one go, but as a series of iteration updates, each adding up to the original approximate values.

The reason for using an ellipsoidal model in MOVE3 is two-fold:

- First of all GNSS/GPS observations can easily be handled in such a model, because they are expressed in Cartesian or ellipsoidal coordinate differences.
- Furthermore, an ellipsoidal model provides a means for the unification of the classical terrestrial horizontal (2D) and vertical (1D) observations.

Consequently an ellipsoidal model is perfectly suitable for the combination of GNSS/GPS and terrestrial measurements.

It is essential in MOVE3 that observations enter the mathematical model in their original form. This means that the observations are not reduced to the ellipsoidal surface, and observation types are not transformed into other types prior to the adjustment. This is of critical importance for the testing; only testing original observations will allow a clear interpretation of possible rejections. Testing of derived observations can make the interpretation complex and diffused. For instance, in MOVE3 the original GNSS/GPS baseline elements (DX,DY,DZ) are used as input and not, as sometimes seen in other software, azimuths and distances derived from the baselines. Thus, in MOVE3 rejections can immediately be traced back to the observations.

Another advantage of working with an ellipsoidal model, with latitude and longitude as coordinate unknowns, is that the map projection is neatly kept out of the adjustment. This advantage can be clarified if we look at the alternative, which is a map projection plane as reference surface in the adjustment. It then becomes necessary to account for the distortion, inherent to map projections, within the adjustment (see paragraph 3.3.1, Purposes and Methods of Projections). Usually these distortions are too complex for the linearised mathematical model to handle. This makes the adjustment using a map projection plane only locally applicable for networks of limited size, whereas the mathematical model in MOVE3 imposes no limits on the size of the network at all.

To summarise the advantages of the ellipsoidal model in MOVE3, one could say that:

- The ellipsoidal model is best suited for the combination of GNSS/GPS and terrestrial measurements.
- The original observations are tested, and not observation derivatives. This allows for a clear link between the testing and the observations.
- The distortion due to the map projection is accounted for by applying map projection formulas before the adjustment. This makes MOVE3 applicable for networks of any size.


### 2.2.2. Observation Types

MOVE3 can handle the following observation types:

- directions [R];
- distances [S];
- zenith angles [Z];
- azimuths [A];
- height differences [DH];
- local coordinates [E], [N], [H];
- shift vector [DE], [DN], [DH];
- GNSS/GPS baselines [DX];
- observed GNSS/GPS coordinates [X];
- 2D geometrical relations [GR]:
- angle between 3 points [AN];
- perpendicular [PD];
- co linearity (3 points on one line) [CL];
- distance point-line [PL];
- parallel lines [PA], including mutual distance [LL];
- perpendicular lines [AL];
- chainage $[\mathrm{CH}]$ and offset [PL];
- double distance [DD];
- tape distance [TD];
- identical [EQ].


## Directions.

In MOVE3 directions are horizontal directions given in GON (centesimal degrees), $D E G$ (sexagesimal degrees) or DMS (degrees, minutes, seconds). A direction is represented by observation type $R$, followed by a digit specifying the series, default $R$. A maximum of 100 series per station is allowed.

## Distances.

In MOVE3 distances are regarded as horizontal or slope, depending on the dimension of the solution and on the availability of zenith angles (see paragraph 2.2.3, Dimension Switch and Observation Type). Distances are given in meters. A distance is represented by observation type $S$, followed by a digit specifying the associated scale factor, default $S 0$. A maximum of 10 scale factors per network is allowed.

## Zenith angles.

In MOVE3 a zenith angle is given in GON (centesimal degrees), DEG (sexagesimal degrees) or $D M S$ (degrees, minutes, seconds). A zenith angle is represented by observation type $Z$, followed by a digit specifying the associated vertical refraction coefficient, default $Z 0$. A maximum of 10 refraction coefficients per network is allowed.

The direction series (RO..R199) have no relationship with scale factors (SO..S9) and refraction coefficients (Z0..Z9). For example: a total station record can consist of R1, S0 and $Z 0$.

## Total station record.

Directions, distances and zenith angles may be combined on one observation record; a so-called total station record. Other allowed combinations are:

- direction and distance;
- direction and zenith angle;
- distance and zenith angle.

The interpretation of these combinations depends on the dimension of the solution and the dimension of the record (see paragraph 2.2.3, Dimension Switch and Observation Type).

## Azimuths.

In MOVE3 an azimuth is a horizontal direction giving the angle between the north direction and the direction to a target. Azimuths are given in GON (centesimal degrees), $D E G$ (sexagesimal degrees) or $D M S$ (degrees, minutes, seconds). An azimuth is represented by observation type $A$, followed by a digit specifying the associated azimuth offset, default $A 0$. A maximum of 10 offsets per network is allowed.

## Height differences.

In MOVE3 height differences are given in meters. When a geoid model is available, the levelled (and/or trigonometric) height differences and orthometric heights are converted to ellipsoidal height differences and heights prior to the adjustment (see paragraph 3.2.3, Geoid and Height Definition). The heights presented after the adjustment are again orthometric. When a geoid model is not available, the known heights and height differences are interpreted as ellipsoidal heights and height differences. No conversion is applied in that case. A height difference is represented by observation type $D H$.

## Local coordinates.

A Local coordinate in MOVE3 is a coordinate set in the selected projection. The observation may consist of $X$ East and $Y$ North coordinates only, height only or a combination of XY and height. The observation type is E for the Easting, N for the Northing and H for the height.

## Shift vector.

A Shift vector in MOVE3 is a vector, coordinate difference in the selected projection. The observation may consist of delta X East and delta Y North coordinates only, height difference only or a combination of delta XY and height difference. The observation type is DE for the delta Easting, DN for the delta Northing and DH for the height difference.

## GNSS/GPS baselines.

In MOVE3 a GNSS/GPS baseline is a 3D vector comprising three Cartesian coordinate differences in GNSS/GPS. (In some receiver post processing software it is allowed to specify a reference systems different from WGS'84. In any case, MOVE3 assumes baselines are given WGS'84.) GNSS/GPS baselines are given in meters, and are represented by observation type $D X$.

## Observed GNSS/GPS coordinates.

In MOVE3 an observed GNSS/GPS coordinate is a 3D Cartesian coordinate in WGS'84. They are useful for approximate determination of the seven transformation parameters between WGS'84 and the user defined local datum. Observed GNSS/GPS coordinates are given in meters, and are represented by observation type $X$.

## Geometrical relations.

The observation types related to more than two stations are captured under the name geometrical relations. Following geometrical relations can be used in MOVE3:

Angle, an arbitrary horizontal angle between three points, expressed in GON (centesimal degrees), $D E G$ (sexagesimal degrees) or DMS (degrees, minutes, seconds). An angle is represented by the observation type AN.

Perpendicular, the perpendicular angle between 3 points ( 100 or 300 gon). MOVE3 will determine, based on the approximate coordinates which angle is applicable. The standard deviation is given in GON, DEG or DMS and the observation type for perpendicular is PD.

Co linearity, the relation that 3 points are located on a straight line, the standard deviation for co linearity is given in meters, the observation type is CL.

Distance point - line, the perpendicular distance of a point to a line (consisting of two other points). The distance is given in meters and the observation type is PL.

Parallelism, two lines are parallel. The two lines each consist of two points. The standard deviation is given in GON, DEG or DMS and the observation type for parallelism is PA.

Distance between two parallel lines, between two parallel lines a distance can be measured. The distance is given in meters and the observation type is LL.

Perpendicular lines, two lines a perpendicular to each other. MOVE3 will determine, based on the approximate coordinates which angle is applicable (100 or 300 gon). The standard deviation is given GON, DEG or DMS and the observation type is AL.

Chainage and offset, a combination of chainage, distance to the beginning of the measurement line, and the offset, the perpendicular distance to this line, given in meters. The observation type for chainage is CH and for offset PL. Chainage and offset can only be used as a combination of both.

Double distance, a station measured from two other stations by tape distances including left right information. The observation type for double distance is DD.

Tape distance, a tape measured distance. The observation type for tape distance is TD.

Identical, can be used when two stations with different names should have identical coordinates. The observation type for identical is EQ.

All geometrical relations containing distances (chainage, offset, distance point to line, distance perpendicular lines, double distance and tape distance) refer to scale factor 0 .

All geometrical relations are pure 2D observations. All distances in geometrical relations are horizontal distances.

### 2.2.3. Dimension Switch and Observation Type

In MOVE3 the dimension of the network solution can be chosen with the Dimension Switch. However, the user cannot simply choose any dimension for his solution. Normally the choice of the dimension of the solution depends on the observations: if stations are connected with observations, which control the horizontal situation as well as the height, a 3D solution is possible. If all stations are only controlled in the height or horizontal situation, a 1D or 2D solution (respectively) must be chosen.

In a 3D solution it is not required to solve for position and height of every station. Depending on the available observations MOVE3 decides which dimension is appropriate for a certain station. This way levelling markers (height differences only) will only be solved in height. Stations that are linked to the network by directions will only be solved in position.

In any case, MOVE3 will try to make full use of the information contained in the observations, within the limitations imposed by the dimension of the solution. Thus, depending on the chosen dimension of the solution, all MOVE3 observation types, or just a subset, contribute to the solution (see table 2.2.1).

| Dimension: | Contributive Observation Type: |
| :--- | :--- |
| 3D | direction <br> horizontal distance <br> slope distance <br> zenith angle <br> azimuth <br> height difference <br> local coordinate (East North Height) <br> shift vector (DEast DNorth DHeight) <br> GNSS/GPS baseline <br> observed GNSS/GPS coordinate <br> geometrical relations |
| 2D | direction <br> horizontal distance <br> slope distance + zenith angle $\rightarrow$ reduced horizontal distance <br> azimuth <br> local coordinate (East North) <br> shift vector (DEast DNorth) <br> GNSS/GPS baseline <br> geometrical relations |
| 1D | slope distance + zenith angle $\rightarrow$ reduced trigonometric height difference <br> height difference <br> local coordinate (Height) <br> shift vector (DHeight) |

table 2.2.1: The relationship between dimension and observation type.
As can be seen in table 2.2.1, a distinction exists between horizontal distances and slope distances. A horizontal distance is obtained by reducing the slope distance between station and target, using the zenith angle. The way the distances, zenith angles (and combinations thereof on the same record) are handled, depends on the dimension of the solution:

- In 3D mode, single distances i.e. distances not combined on the same record with a zenith angle, are regarded as horizontal distances if the dimension of the total station record is set to 2 D and as a slope distance if the dimension of the record is set to 3D. A distance combined with a zenith angle is regarded as a slope distance. If the dimension of the record is set to 2D the slope distance will be reduced to a horizontal distance before the adjustment. In this case the zenith angle will not be used as a separate observation in the adjustment. This record will not contribute to solving the height component of the station and the target.
- In 2D mode only horizontal distances can be used. A distance combined with a zenith angle is reduced to a horizontal distance prior to the adjustment. Single zenith angles are disregarded in the 2D solution.
- In 1D mode, single distances, single zenith angles and combinations of distances and deselected zenith angles are disregarded. A distance combined with a zenith angle is regarded as a slope distance. This combination is reduced to a trigonometric height difference prior to the adjustment.

See table 2.2.2 below, for a summary of these cases.

| Dimension |  | Observations | Result |
| :---: | :---: | :---: | :---: |
| network | record |  |  |
| 3D | 3D | S + Z | slope distance + zenith angle |
|  | 3D | S | slope distance |
|  | 2 D | S + Z | horizontal distance (reduced) |
|  | 2D | S | horizontal distance |
|  | 1D | S + Z | height difference (reduced) |
| 2D | 3D | S + Z | horizontal distance (reduced) |
|  | 3D | S | not used |
|  | 2 D | S + Z | horizontal distance (reduced) |
|  | 2D | S | horizontal distance |
| 1D | 3D | S + Z | height difference (reduced) |
|  | 1D | S + Z | height difference (reduced) |

table 2.2.2: Handling of distance - zenith angle combination.
Whenever zenith angles are part of the input, the influence of refraction can be taken into account by the refraction coefficients. For 3D solutions the coefficients can be solved as part of the adjustment. However, for 2D and 1D solutions these coefficients remain fixed on a user defined value.

### 2.2.4. Combining Observations

In MOVE3 all observation types may be combined freely, as long as the combination does not lead to an ill-posed adjustment problem. This also applies for the combination of terrestrial and GNSS/GPS observations; any combination of GNSS/GPS observations and terrestrial observations, bearing in mind the stipulation above, is allowed. For example, a GNSS/GPS network strengthened by a few terrestrial observations can easily be handled by MOVE3.

Depending on the observation types included in the network, a 3D, 2D or 1D solution is feasible. Apart from the dimension, a number of other elements should to be taken into account:

- the type of network (terrestrial and/or GPS);
- the type of known coordinates (in a map projection plane or ellipsoidal/cartesian);
- the projection type (Transverse Mercator, Lambert or Stereographic);
- the reference ellipsoid (local or global).

The reference ellipsoid is the key element here. Since the mathematical model in MOVE3 is ellipsoidal, it is always necessary to choose a reference ellipsoid. This means that also in the simple case of a levelling network comprising observed height differences and known heights, an ellipsoid must be specified. The case in which the reference ellipsoid is the WGS'84 ellipsoid is regarded as a special case in MOVE3, since WGS'84 plays an important role as a global datum.

The elements mentioned above are included in 9 different cases, presented in table 2.2.3. The table can be helpful when preparing input for a MOVE3 adjustment. The last two columns indicate the coordinate type of the known stations, and the file type these stations should be in. The coordinate and file type are based on the network type, projection type and ellipsoid (columns 2, 3 and 4). In table 2.2.3 'global' explicitly means WGS'84 or WGS'72, while 'local' means any ellipsoid other than the WGS'84 or WGS'72 ellipsoid, i.e. the ellipsoid of a local datum such as the Bessel ellipsoid.

In MOVE3 two separate files contain coordinates: the Terrestrial COordinates are in the TCO file, and the GNSS/GPS COordinates file are in the GCO file. Depending on the type of network one of the two, or both files are part of the input:

- A TCO file is needed when the network includes terrestrial observations, or when a local ellipsoid is chosen. This covers all cases except case 5. In case a projection is defined or in case no projection is defined but a local ellipsoid is chosen, the TCO file must include the known stations.
- A GCO file is required when the network includes GNSS/GPS observations or when a global ellipsoid is chosen. This covers all cases except the cases 1 and 3. When a global ellipsoid is chosen, the GCO file must include the known stations.

| Case | Network | Projection | Ellipsoid | Known stations |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Coord type | File |
| 1 | TER | None | local | ( $\varphi, \lambda, \mathrm{h})$ | TCO |
| 2 | TER | None | global | ( $\varphi, \lambda, \mathrm{h}$ ) or (X,Y,Z) | GCO |
| 3 | TER | Mer/Lam/Ster | any | (E,N,h) | TCO |
| 4 | GPS | None | local | ( $\varphi, \lambda, \mathrm{h})$ | TCO |
| 5 | GPS | None | global | ( $\varphi, \lambda, \mathrm{h}$ ) or (X,Y,Z) | GCO |
| 6 | GPS | Mer/Lam/Ster | any | (E,N,h) | TCO |
| 7 | TER+GPS | None | local | ( $\varphi, \lambda, \mathrm{h})$ | TCO |
| 8 | TER+GPS | None | global | ( $\varphi, \lambda, \mathrm{h})$ or (X,Y,Z) | GCO |
| 9 | TER+GPS | Mer/Lam/Ster | any | (E,N,h) | TCO |
| ( $\varphi, \lambda, \mathrm{h})=$ ellipsoidal coordinates (latitude, longitude, height) <br> ( $\mathrm{E}, \mathrm{N}, \mathrm{h}$ ) = map projection coordinates (X East, Y North, height) <br> ( $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ ) = Cartesian coordinates |  |  |  |  |  |

table 2.2.3: The 9 cases for processing terrestrial and/or GPS networks.
The coordinate type in the TCO file, ( $\mathrm{E}, \mathrm{N}, \mathrm{h}$ ) or ( $\varphi, \lambda, \mathrm{h}$ ), is controlled by the PROJECTION parameter in this file. In the GCO file the coordinate type, $(\varphi, \lambda, \mathrm{h})$, ( $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ ) or ( $\mathrm{E}, \mathrm{N}, \mathrm{h}$ ), is controlled by the COORDINATES parameter (see MOVE3 under GPS coordinate file(GCO)).

In order to facilitate the interpretation of table 2.2.3 consider the following examples:

## Example 1:

The network in this example consists of a combination of terrestrial and GNSS/GPS observations. Known stations are available for this network in some map projection. This is case 9 in table 2.2.3:

- a combined terrestrial/GPS network;
- a map projection is specified;
- the projection is combined with a specific ellipsoid (e.g. the RD projection and the Bessel 1841 ellipsoid).

The known and approximate coordinates for TER stations must be given in the TCO file in the specified map projection. Approximate coordinates for GNSS/GPS stations must be provided in the GCO file.

## Example 2:

A simple 1D levelling network has to be adjusted. In order to link the network to the reference ellipsoid, all stations must be given in 3D (approximate) coordinates. This is a consequence of the ellipsoidal model used in MOVE3. Hence, for a levelling network it is necessary to enter known heights and approximate coordinates as ( X East, Y North) or as (latitude, longitude). Presuming that the necessary approximate coordinates are scaled from a topographic map in some map projection case 3 in table 2.2.3 applies:

- the network is terrestrial;
- a map projection is specified;
- the projection is combined with a specific ellipsoid (e.g. the RD projection and the Bessel 1841 ellipsoid).

The known and approximate coordinates must be given in the TCO file in the specified map projection. A GCO file is not necessary.

## Example 3:

Consider the case of the free network adjustment of a GNSS/GPS network. At this stage the main concern is the correctness of the GNSS/GPS observations, rather than the computation of the final adjusted coordinates. Therefore a map projection is not yet specified. The reference ellipsoid is then by definition the GNSS/GPS reference ellipsoid WGS'84. This is case 5 in table 2.2.3:

- a GNSS/GPS network;
- projection is none;
- the reference ellipsoid is WGS'84.

The known and approximate coordinates must be given in the GCO file. A TCO file is not necessary. Note that this example is valid for both 3D and 2D.

Although a variety of cases may occur, it is most likely in practice that the user has known stations in some map projection at his disposal. Then, by definition, the known stations must be given in the TCO file. In case the network includes GNSS/GPS observations, approximate coordinates for the GNSS/GPS stations must be given in the GCO file. Thus, the ellipsoidal model is completely concealed, and the user's only concern is the selection of the proper map projection and ellipsoid.

### 2.2.5. More information...

More information about using the MOVE3 network adjustment software can be found in the on-line help utility of the MOVE3 Windows user interface. To request Help, use one of the following methods:

- From the $\underline{H e l p}$ menu, choose a Help command;
- Press F1;
- Choose the Help button available in the dialog box. This method gives you quick access to specific information about the dialog box.


## 3. Geodetic Concepts

### 3.1. Introduction

This part of the manual introduces the user to some of the theoretical fundamentals of MOVE3. It is presumed that the user possesses some basic knowledge of surveying and adjustment computations. A complete presentation of all theoretical aspects is beyond the scope of this manual. The reader is referred to the literature list in paragraph 5.2, Literature List.

The mathematical model in MOVE3 is rigorously ellipsoidal. The coordinate unknowns are latitude, longitude and height. Consequently it is necessary to select an ellipsoid as a reference system in the adjustment. In addition, in many cases a map projection is required to relate the input Easting and Northing to the internal ellipsoidal unknowns. Therefore, the paragraphs 3.2 and 3.3 deal with reference systems and map projections. Another reason for discussing reference systems is the vital role of the World Geodetic System 1984 (WGS'84) in GNSS/GPS positioning.

Paragraph 3.5, is dedicated to GNSS/GPS. GNSS/GPS is now an important measurement tool in many surveys. Hence a separate chapter of this manual is dedicated to this subject. Of course this chapter will mainly focus on those aspects of GNSS/GPS, which relate to MOVE3, namely relative positioning using phase measurements. Special attention will be paid to GNSS/GPS in control networks. GNSS/GPS allows for a different approach, especially when designing networks, as compared to the classical approach with terrestrial observations. Last but not least, the combination of GNSS/GPS and terrestrial networks is discussed.

Paragraph 3.5, Detail Measurements treats the processing of detail measurements in the adjustment. Specific features as geometric relations, idealisation precision and offset measurements are explained.

### 3.2. Reference Systems

The mathematical model in MOVE3 is rigorously ellipsoidal. This means that MOVE3 internally uses ellipsoidal coordinates. Consequently the user has to select an ellipsoid as reference surface in the adjustment. Therefore this chapter deals with reference systems in general, and the vital role of the World Geodetic System 1984 (WGS'84) in GNSS/GPS positioning in particular.

### 3.2.1. General

By positioning we understand the determination of the coordinates of a point, with respect to a well-defined coordinate system. This immediately illustrates the fundamental role of coordinate reference systems.

The two most commonly used coordinate systems for fixing positions in threedimensional space are (figure 3.2.1):

figure 3.2.1

## The ellipsoidal coordinate system.

The reference is a mathematically defined curvilinear coordinate system: the ellipsoid. The advantage of using an ellipsoid is that its surface roughly corresponds to the surface of the earth, which makes the interpretation of ellipsoidal coordinates easier. A point P in geodetic ellipsoidal coordinates is given by:

| longitude | $\lambda_{P}$ |
| :--- | :--- |
| height | $h_{P}$ |

## The Cartesian coordinate system.

A point $P$ in such a system is fixed by means of three distances to three perpendicular axes $\mathrm{X}, \mathrm{Y}$ and Z . The axes usually define a right-handed system. The XY plane is in the equatorial plane. The positive X -axis is in the direction of the Greenwich meridian. The positive Z-axis, perpendicular to the XY plane points towards the North Pole. The coordinates of a point $P$ are given as: $X_{p}, Y_{p}, Z_{p}$. $A$ Cartesian system is very suited for representation of relative positions, such as GNSS/GPS baselines.

There is a direct relationship between the two coordinate systems. Hence, ellipsoidal coordinates can easily be transformed into Cartesian coordinates and vice versa.

The ellipsoid itself is defined by its semi major axis $a$ and semi minor axis $b$, or by its semi major axis and flattening $f$. There is a simple relationship between the axes and the flattening:

$$
f=(a-b) / a
$$

The positioning of the ellipsoid requires six more parameters to eliminate the six degrees of freedom, i.e. the six ways (three translations, three rotations) in which the ellipsoid can move relative to the earth. The task of appropriately positioning a reference ellipsoid is known as the establishment of a datum.

In classical geodesy the reference ellipsoid is used as a horizontal datum only. Heights are given with respect to mean sea level or, more precisely, with respect to the geoid. In this context the geoid serves as a vertical datum (see paragraph 3.2.3, Geoid and Height Definition).

### 3.2.2. Global and Local Systems

A distinction can be made between global and local reference systems (figure 3.2.2). The GNSS/GPS reference system WGS' 84 for example, is a global reference system. Its origin is supposed to coincide with the earth's centre of gravity $\mathbf{C}$, and its Z-axis is supposed to coincide with the earth's rotational axis. WGS'84 is the latest in a series of earth-centred, earth-fixed (ECEF) coordinate systems (WGS'60, WGS'66, WGS'72). Each of these systems is based on updated information, and therefore successively more accurate. The general importance of WGS' 84 is that it provides a means for relating positions in various local reference systems.

figure 3.2.2
Not all reference ellipsoids are geocentric (earth-centred) ellipsoids. Many geodetic reference ellipsoids are selected so as to approximate the earth's surface only in a certain area (country, group of countries, continent). For this purpose the ellipsoid is appropriately positioned within the earth, and consequently in most cases nongeocentric. The origin of the ellipsoid is situated in $\mathbf{E}$, and not in $\mathbf{C}$ (figure 3.2.2).

Examples of datums based on 'local' non-geocentric ellipsoids are European Datum 1950, North American Datum 1927, Dutch Datum, Ordnance Survey UK 70 and so on. These datums are based on different ellipsoids: European Datum 1950 is based on the International Hayford 1924 ellipsoid, North American Datum 1927 is based on Clarke 1866, Dutch Datum is based on Bessel 1841, Ordnance Survey UK 70 is based on Airy.

The 'local' ellipsoids also play a role in the definition of national coordinate systems. These are defined by a reference ellipsoid and a map projection (see chapter 3.3, Map Projections). The Dutch RD-coordinates, for example, are derived by transforming coordinates on the Bessel ellipsoid to a projection plane using the Stereographic projection formulae.

### 3.2.3. Geoid and Height Definition

The geoid can be described as the surface, which best approximates mean sea level over the whole earth (figure 3.2.2). It is a gravity equipotential surface, i.e. a surface perpendicular to the lines of gravitational force. The earth's gravity field is rather complex, resulting in an irregular though smooth geoid. The determination of the geoid is based on terrestrial and sea gravity measurements and on information provided by satellites. This information originates from the analysis of satellite orbits and satellite altimeter measurements to the ocean surface. Because the availability and quality of this data varies over the globe, the geoid is only approximately known.

The height defined in paragraph 3.2.1, General, is an ellipsoidal height $\mathbf{h}$, a height above a mathematically defined ellipsoid. More commonly used are orthometric heights $\mathbf{H}$. These heights are based on the geoid as vertical datum. Ellipsoidal heights have gained importance since the introduction of satellite positioning. Before the introduction of satellites, heights were measured solely by levelling instruments along the vertical with the geoid as null-reference. Because orthometric heights are indirectly based on gravity measurements they posses a practical quality that ellipsoidal do not posses: they tell us which way the water flows.

Ellipsoidal heights h can be converted into orthometric heights H by means of geoid heights or undulations N (figure 3.2.3). The following simple relationship holds:

$$
h_{A}=H_{A}+N_{A}
$$

The geoid heights, with respect to the WGS'84 ellipsoid, range from about -100m south of India to about +65 m around Iceland. The standard deviation for the geoids heights varies per region, depending on the available gravity data. More important however, is the precision of the geoid height differences, $N_{A}-N_{B}$. In areas with a dense net of gravity measurements, a precision of about 2 to 4 ppm of the distance is feasible. In case no geoid information is available, the levelled heights are just substituted by ellipsoidal heights. The error thus introduced will then vary depending on whether the area is flat or mountainous.

figure 3.2.3
The absolute error in the height differences may be up to 1 m over a 10 km distance. This error is mainly systematic, because the geoid is usually relatively smooth over the project site. Therefore the influence of the change in the geoidellipsoid separation will substantially be "absorbed" (and thus accounted for) by the GNSS/GPS transformation parameters. Errors because of the unmodelled undulations are within the decimeter level.

The correct procedure, necessary for critical applications, is to convert orthometric heights and height differences into ellipsoidal heights and height differences before the adjustment, using the available geoid heights. After the adjustment the computed ellipsoidal heights are vice versa converted to orthometric heights.

### 3.2.4. Datum Transformations

GNSS/GPS is indeed a global positioning system and is consequently based on a global datum. Characteristic for such a global or world datum is (see paragraph 3.2.2, Global and Local Systems):

- its origin is supposed to coincide with the earth's centre of mass;
- its Z-axis is supposed to coincide with the earth's rotational axis.

As seen earlier in this chapter, there are also local datums based on an ellipsoid, which best fits the earth's surface over a specific area. For the transformation of the coordinates from one datum to another, it is necessary to have knowledge about the relative positions of the various datums. A datum transformation can be carried out when the seven parameters of a 3D similarity transformation are known (figure 3.2.4): 3 translation parameters ( $T_{x}, T_{y}, T_{z}$ ), 3 rotation parameters ( $\alpha, \beta, \gamma$ ) and 1 scale factor.

figure 3.2.4
Usually the rotation and scale differences between two datums are very small; in the order of a few arc seconds or a few ppm. The translation parameters, giving the shift between the two origins, can have values in the order of several hundreds of meters. For instance, the shift between the Dutch datum (Bessel 1841 ellipsoid) and the WGS'84 datum is approximately $593 \mathrm{~m}, 26 \mathrm{~m}$ and 478 m , in X-, Y-, and Zdirection respectively.

When GNSS/GPS observations are to be included in a local datum, in which e.g. the known stations are given, a transformation is necessary. For GNSS/GPS baselines, the translation between WGS'84 and the local datum need not be solved. Thus in the adjustment only 4 transformation parameters remain. In MOVE3 the 4 transformation parameters are solved as part of the adjustment. The user therefore does not have to enter these parameters. As a consequence these parameters only have a local significance, and cannot be presumed valid for areas beyond the extent of the pertaining network.

Besides solving the transformation parameters it is also possible to keep the transformation parameters fixed or weighted fixed.

[^0]
### 3.3. Map Projections

After a short general introduction, this chapter describes the most commonly used map projections: the Transverse Mercator, the Lambert and the Stereographic projection. Map projections are applied in MOVE3 to relate the entered X East and Y North coordinates to the internally used ellipsoidal coordinates.

### 3.3.1. Purposes and Methods of Projections

In surveying it is often more convenient to work with rectangular coordinates on a plane, than with ellipsoidal coordinates on a curved surface. Map projections ( $F_{m p}$ ) are used to transform the ellipsoidal latitude and longitude into rectangular X East and Y North, and vice versa:


$\triangle$
The orientation of the planar coordinate system is often a source of confusion in map projections. In some countries, for instance in Germany, the positive X-axis points towards the north, while the positive $\mathbf{Y}$-axis points toward the east. In other countries, for instance in the Netherlands, the situation is reversed. To avoid confusion, the $\mathbf{X}$ - and $\mathbf{Y}$-coordinates are often referred to as Easting and Northing.

The representation of the curved ellipsoidal surface of the earth on a plane will inevitably result in a distortion of geometric elements. With respect to this distortion, map projections are usually subdivided in conformal, equidistant and equivalent projections. Conformal projections preserve the angle of any two curves on the ellipsoidal surface. Because of this property, only conformal projections are employed for geodetic purposes. Equidistant (equality of distances) and equivalent (equality of areas) projections are sometimes used in other disciplines, such as cartography.

Another subdivision of map projections is based on the projection surface.
Projections can then be subdivided into cylindrical, conical and plane, depending on whether the projection surface is a cylinder, a cone or a plane (figure 3.3.1).

figure 3.3.1
The orientation of the projection surface with respect to the ellipsoid can be adapted depending on the area of interest. Especially for these so-called oblique cases, a sphere is introduced as an intermediate between the ellipsoid and the plane. In these cases the projection formulae represent a relation between spheroidal and plane coordinates.

The three most commonly used projections (and their projection surface) are:

- the Transverse Mercator projection (cylinder);
- the Lambert projection (conical);
- the Stereographic projection (plane).

Different countries use different projections. This fact is dictated by the desire to keep the distortion within reasonable limits, given the shape of the area (country) of interest. Thus countries having their largest extension in east-west direction generally have adopted the Lambert projection, since the scale distortion in eastwest direction is minimal for this projection. The Lambert projection has been adopted in Belgium, France, and Denmark. Some states of the US use the Lambert projection as well (others use Transverse Mercator).

For the same motivation countries with a predominant north-south extension generally have adopted the Transverse Mercator projection. The general Mercator projection is a cylinder projection with the axis of the cylinder directed North-South. In the Transverse Mercator projection the axis of the cylinder is in the plane of the equator. The projection is used in Great Britain, Australia, Canada and in the former USSR. In German-speaking countries the Transverse Mercator projection is called the Gauss-Krüger projection. The Mercator projection is known worldwide through the Universal Transverse Mercator (UTM) projection. The UTM projection covers the complete globe in 60 zones of 6 degrees.

In the Netherlands an oblique Stereographic projection has been adopted, as well as in other roughly circular shaped countries such as Poland and Hungary. The Dutch national coordinate system, based on the Stereographic projection, is called the 'RD' system.

Examining the grid of meridians and parallels on the map can identify projections:

- In the Transverse Mercator projection the earth's equator and the central meridian, the tangent cylinder-ellipsoid, are projected as straight lines. Other meridians and parallels are projected as complex curves.
- The Lambert projection pictures parallels as unequally spaced arcs of concentric circles. Meridians are projected as equally spaced radii of the same circles.
- The polar Stereographic projection pictures parallels as concentric circles, and meridians as straight lines radiating at true angles from the polar centre of projection. The oblique alternative of this projection pictures all parallels and meridians as circles. Exceptions are the meridian of longitude of origin, and the parallel opposite in sign to the parallel of latitude of origin. The latter two are shown as straight lines.

Projections are defined by a number of parameters. The interpretation of these parameters may differ for the various projections. The parameters are reviewed in the next paragraphs.

### 3.3.2. The Transverse Mercator Projection

The Transverse Mercator projection is based on the following parameters:

Central Meridian (CM): | Normally the meridian running through the centre of the area of interest, |
| :--- |
| defining together with the latitude of origin the origin of the plane coordinate |
| system. There is no scale distortion along the central meridian. |

Longitude of Origin: $\quad$| See Central Meridian. |
| :--- |

Latitude of Origin: $\quad$| Normally the parallel running through the centre of the area of interest, |
| :--- |
| defining together with the central meridian the origin of the plane coordinate |
| system. |

Scale Factor: $\quad$| The scale factor is constant along the central meridian. The value assigned |
| :--- |
| to the scale factor along this line is often slightly smaller than 1, so that the |
| overall scale of the map is more neatly correct. |

False Easting: $\quad$| For some projections a False Easting is introduced to prevent negative |
| :--- |
| coordinates. A False Easting is simply a large positive value that is added to |
| the original Easting. In some cases the False Easting is assigned a specific |
| value, making Eastings immediately distinguishable from Northings. |

False Northing: $\quad$| For some projections a False Northing is introduced to prevent negative |
| :--- |
| coordinates. A False Northing is simply a large positive value that is added |
| to the original Northing. In some cases the False Northing is assigned a |
| specific value, making Northings immediately distinguishable from Eastings. |

### 3.3.3. The Lambert Projection

There are two ways to define the Lambert projection, depending on the number of standard parallels (see figure 3.3.2):

- In case of just one standard parallel $\varphi_{0}$, the scale factor $m_{0}$ along this parallel will have a value slightly smaller than 1 .
- In case of two parallels $\varphi_{1}$ and $\varphi_{2}$, the scale factor is equal to 1 along both parallels.

The cases are related: given one standard parallel with scale factor $m_{0}$, the two parallels $\varphi_{1}, \varphi_{2}$, along which the scale factor is 1 , can be deduced and vice versa.

figure 3.3.2
The Lambert projection is based on the following parameters:

Standard Parallel(s): Two standard parallels represent the intersecting circles of the cone and the ellipsoid. A single standard parallel is usually not an intersecting circle, but represents a circle in between the two circles that intersect the cone.
Longitude of Origin: Normally the meridian running through the centre of the area of interest, defining together with the latitude of origin the origin of the plane coordinate system.

Latitude of Origin: Normally the parallel running through the centre of the area of interest, defining together with the longitude of origin the origin of the plane coordinate system.

Scale Factor: Sometimes a scale factor along the single standard parallel is introduced with a value slightly smaller than 1 , so that the overall scale of the map is more neatly correct.

False Easting: For some projections a False Easting is introduced to prevent negative coordinates. A False Easting is simply a large positive value that is added to the original Easting. In some cases the False Easting is assigned a specific value, making Eastings immediately distinguishable from Northings.

False Northing: $\quad$| For some projections a False Northing is introduced to prevent negative |
| :--- |
| coordinates. A False Northing is simply a large positive value that is added to |
| the original Northing. In some cases the False Northing is assigned a specific |
| value, making Northing immediately distinguishable from Castings. |

### 3.3.4. The Stereographic Projection

The Stereographic projection is based on the following parameters:

| Longitude of Origin: | Longitude of the central point (the tangent point of ellipsoid and plane) of the <br> projection, usually in the centre of the area of interest. <br> Latitude of the central point of the projection, usually in the centre of the area <br> of interest. |
| :--- | :--- |
| Latitude of Origin: | The scale factor at the central point of the projection, as defined by the latitude <br> and longitude of origin. The value assigned to the scale factor at this point is <br> often slightly smaller than 1, so that the overall scale of the map is more neatly <br> correct. |
| Scale Factor: | For some projections a False Easting is introduced to prevent negative <br> coordinates. A False Easting is simply a large positive value that is added to <br> the original Easting. In some cases the False Easting is assigned a specific |
| False Easting:value, making Fastings immediately distinguishable from Northing. |  |
| False Northing: $\quad$For some projections a False Northing is introduced to prevent negative <br> coordinates. A False Northing is simply a large positive value that is added to <br> the original Northing. In some cases the False Northing is assigned a specific <br> value, making Northing immediately distinguishable from Easting. |  |

### 3.3.5. The Local (Stereographic) Projection

The Local (Stereographic) projection is useful when the network coordinates are given in your own local coordinate system. MOVE3 uses a stereographic projection with the following default values for the parameters:

Longitude of Origin: $\quad 0^{\circ}$
Latitude of Origin: $\quad 0^{\circ}$
Scale Factor: $\quad 1.0$
False Easting: 0 m
False Northing: 0 m

## $\triangle$ <br> The user is free to change these values.

### 3.4. GNSS/GPS

GNSS/GPS is rapidly becoming the major measurement tool in many surveys. Therefore in the next few paragraphs the aspects of GNSS/GPS, which relate to MOVE3, are highlighted. Special attention will be paid to GNSS/GPS in control networks. GNSS/GPS allows for a different approach, especially in designing these networks, as compared to the classical approach with terrestrial observations. In addition, the final paragraph discusses the combination of GNSS/GPS and terrestrial networks.

### 3.4.1. General

GNSS/GPS can be used for various different purposes, from real time navigation to high precision relative positioning. In the latter case GNSS/GPS has a number of advantages over the traditional land surveying methods:

- the line of sight between station and target is no longer a requirement;
- GNSS/GPS observations (baselines) can span large distances;
- observations have a high precision;
- the measurement process is quick and efficient.

The processing of GNSS/GPS observations can be divided into two steps. In the first step the "raw" GNSS/GPS observations, stored in the GNSS/GPS receiver, are processed to get WGS'84 coordinates or WGS'84 coordinate differences (baselines). The processing is usually done with GNSS/GPS post-processing software supplied by the GNSS/GPS receiver manufacturer. The GNSS/GPS baselines are the input for the second step, the MOVE3 adjustment. GNSS/GPS is a global system therefore GNSS/GPS observations are expressed in the world-wide WGS'84 system (World Geodetic System 1984). To get local coordinates from GNSS/GPS observations a transformation is required (see paragraph 3.2.4, Datum Transformations).

### 3.4.2. GNSS/GPS Observations

In GNSS/GPS positioning a distinction can be made between absolute and relative positioning:

- By absolute positioning (figure 3.4.1) we understand the determination of the absolute coordinates of a point on land, at sea or in space with respect to a welldefined coordinate system, e.g. WGS'84. A major disadvantage of absolute positioning is the distortion of GNSS/GPS signals by the controllers of the GNSS/GPS system (Selective Availability) and distortion by atmospheric conditions. These effects influence the computed receiver's position. Therefore accurate absolute positioning is not possible with this technique. Accuracy can be improved by relative measurements (DGPS).

figure 3.4.1

In relative positioning (figure 3.4.2) the position of a point is determined with respect to another point. By processing the signals of both receivers simultaneously most of distortions can be eliminated. The result of the processing, the coordinate difference or baseline, is the input for MOVE3.

figure 3.4.2

### 3.4.3. GNSS/GPS in Control Networks

A control network is a set of fixed points with coordinates attached to them. The aim of a control network is to allow the positioning of topographic features with respect to the fixed points. Thus a geodetic control network is the geometric reference of every Geographic Information System or any other system, application or tool requiring position information.

In point of fact, a GNSS/GPS baseline is just one in the list of observation types including directions, distances, zenith angles, azimuths and height differences (see paragraph 2.2.2, Observation Types). However, GNSS/GPS possesses some features, which require a different approach when using GNSS/GPS in control networks:

- The line of sight between adjacent network stations is no longer a necessity. This, added to the fact that there is practically no limit to the distance between receivers, provides the surveyor with an enormous amount of freedom when designing a network.
- GNSS/GPS is a 3D-measurement technique. The strictly applied, though artificial distinction between horizontal networks and height networks no longer holds.
- A characteristic of GNSS/GPS is that all coordinates and coordinate differences are given in the same unique world-wide reference system. This presents a problem when GNSS/GPS observations are linked to existing known stations given in some local coordinate system. In such cases the parameters of a similarity transformation have to be solved (see paragraph 3.2.4, Datum Transformations).


### 3.4.4. GNSS/GPS stochastic model

As a result of the GNSS/GPS baseline processing besides the coordinate differences also a $3 \times 3$-variance matrix for the coordinates is computed (see paragraph 4.2.3, Stochastic Model). This variance matrix can be used in MOVE3 for the precision of the baseline. In most cases however the computed standard deviations are too optimistic. This may cause rejections in all of the baselines. MOVE3 contains two tools to solve this problem. The first is to scale the standard deviations of the baselines. A second possibility is to use an absolute and relative standard deviation per baseline. The relative part is usually expressed in ppm (parts per million of the baseline length). In this case the correlation between the components of the baseline is ignored.

### 3.4.5. GNSS/GPS and Heights

Height differences measured with GNSS/GPS are always ellipsoidal height differences. To convert ellipsoidal height differences to orthometric height differences a correction for the geoid needs to be applied (see paragraph 3.2.3, Geoid and Height Definition). The precision of the height differences is usually rather limited, thus influencing the total precision of GNSS/GPS height determination.

Even when the geoid is not taken into consideration, GNSS/GPS cannot compete with levelling.

### 3.5. Detail Measurements

The target of detail measurements is to establish the mutual position of object in the field. Strictly spoken the detail measurements do not differ from control measurements, but a number of characteristics are specific for detail measurements. Detail networks usually contain a lot of observations, influencing the performance of an integral adjustment. Due to the fact that the objects measured can have different nature, the precision of idealisation needs to be taken into account and some of the measurements use a prism offset. Geometrical relations are sometimes used in detail networks besides total station and GNSS/GPS observations. These typical aspects of detail measurement are described in this paragraph.

### 3.5.1. Geometrical relations

MOVE3 uses the term geometrical relations as a collection of observation types that are typically used in detail measurements. A specific feature of geometrical relations is that they refer to three or four points, contrary to other observation types. The geometrical relations that contribute to a MOVE3 adjustment will be described one by one:

## Angle

This is an arbitrary angle between three points. In practice usually a perpendicular angle (100 or 300 gon) will be used. The sequence of the stations is important for the definition of the angle:


Figure 3.5-1, Angle
An angle in MOVE3 is defined as a right turning angle (clockwise).
The observation type angle is also used to import steel tape series from a SFN file. A steel tape series is a combination of straight or right angles and distances.

## Perpendicular

The observation type perpendicular only states that three point are perpendicular, the direction of the angle is not specified. When using this observation type it is important that sufficiently accurate approximate coordinates are available for the three points. The information perpendicular cannot be used to compute approximate coordinates. In the adjustment, MOVE3 will determine whether the angle should be 100 or 300 gon, based on the approximate coordinates.

## Co linearity

The observation type co linearity states that three points are located on a straight line:


Figure 3.5-2, Co linearity

The ' $A t$ ' station is collinear with the line between 'From' and 'To'. MOVE3 considers co linearity as a distance of a point to a line, in this case the distance is 0 meter. The standard deviation for co linearity is given in meters.

## Distance between point and line

This is the perpendicular distance of a point ('At') to a line ('From' - 'To').


Figure 3.5-3, Distance point - line
The observation has a negative value if the point, as seen from the 'From' station is on the right side of the line. To be able to properly measure the distance between the point and line, the projection of the point on the line is must also be determined. In the total standard deviation the determination of the projection (auxiliary point) is taken into account.

## Parallelism

Indicates that two line elements are parallel. Besides parallelism the mutual distance can be specified.

## Perpendicular lines

Indicates that two line elements are perpendicular. This relation is mainly used for buildings.

## Chainage-offset

MOVE3 also supports chainage and offset measurements. The measured point is projected (auxiliary point) on the measurement line.


Figure 3.5-4, Chainage-offset

## Double distance

If a station has been measured from two other stations by tape distances, these observations can be included in MOVE3 as a double distance. The combination of the two distances and the left right information uniquely determines the new station.


Figure 3.5-5, Double distance

## Tape distance

A tape measured distance can be entered in MOVE3 as a tape distance. A combination of angles and tape distances form tape measurement series.

## Identical

Two stations can be identified as identical using this observation type.

### 3.5.2. Precision of idealisation

It is important in detail adjustments to take into account the precision of idealisation. The precision of idealisation is the precision of identifying a point in the field.
Corners of buildings are easy to identify in the field, but it is rather difficult to identify the centre or the side of a ditch. The precision of idealisation is independent of the type of measurement, it is a feature of the measured point (object).

To specify the precision of idealisation, use can be made of the classes in Table 3.51.

| Class | Classes idealisation | Standard <br> deviation <br> idealisation |
| :--- | :---: | :---: |
| Benchmark, wall (hard topography) | $0.00-0.02 \mathrm{~m}$ | 0.02 m |
| Pavement, street furniture, drain hole | $0.01-0.03 \mathrm{~m}$ | 0.03 m |
| Fence | $0.02-0.05 \mathrm{~m}$ | 0.05 m |
| Hedge, drain | $0.05-0.10 \mathrm{~m}$ | 0.10 m |
| Ditch | $0.10-0.20 \mathrm{~m}$ | 0.20 m |
| Non classified points | $>0.20 \mathrm{~m}$ | 0.32 m |

Table 3.5-1: Classes for precision of idealisation
The specified precision of idealisation is used to compute the total standard deviation of observations, similar to the centring error. The precision of idealisation can also be used to specify a different centring error for each individual station.

### 3.5.3. Eccentric measurement

If the prism cannot be located at the detail point to be measured, an offset must be measured by tape. The prism can be positioned fore or after or left or right of the detail point. The distance of the detail point to the prism is measured by tape. Prior to the adjustment the eccentric measures will be used to compute derived observations. To compute the total standard deviation the standard deviation of the tape measurement and the determination of the correct location of the prism (perpendicular) are taken into account.


Figure 3.5-6, Eccentric measurement

For more information on detail adjustments please refer to HTW 1996 (in Dutch), chapter 6, Detailmeting.

## 4. Quality Control

### 4.1. Introduction

It is important for a surveyor, as for any other professional, to assess and control the quality of his work. The reasons for this are obvious:

- the work will have to meet certain requirements, so it is essential to be able to assess whether the requirements are met;
- when for some structural (bad design) or incidental reason (observation error) the requirements are not met, tools must be available to improve the situation.

Knowing about the costly consequences of ill-designed networks or undetected errors, especially when these deficiencies are discovered in an advanced stage of the survey, it is clear that quality control will save both time and money.

Quality control has gained importance in survey work due to the instrumental developments in recent years. Modern total stations and GNSS/GPS receivers are capable of producing a bulk of survey data, thus demanding efficient tools for assessing their sufficiency and accuracy.

Besides instrumental developments, also developments in survey software have opened the way for an increase in the application of quality control. Not long ago the quality assessment of survey networks by means of reliability parameters and statistical testing was reserved for specialists able to operate complex computer systems. Nowadays by the introduction of easy to operate PC based software packages, such as MOVE3, quality control can reach professionals on all sorts of different levels.

In this part of the manual quality control for geodetic networks is introduced by means of a review of (the parameters of) precision and reliability, according to the 'Delft Method':

- The control of the propagation of random errors, present in the observations, into the coordinates is measured in terms of precision.
- The identification of gross errors and/or biases in the observations and the control over the sensitivity of the data to these errors and biases is measured in terms of reliability.

The descriptions in this chapter are presented at a conceptual level, intended to give the reader a basic understanding of the process. More information can be found in the literature listed in Appendix B.

### 4.1.1. Adjustment, Precision, Reliability and Testing

From observations carried out in the field, the surveyor will have to compute an end result: the coordinates. When redundant observations are available, as it should be, a strategy has to be chosen so as to get a unique and optimal solution. In geodesy, this strategy usually is the least squares adjustment. It is based on the following criterion: the sum of squares of the observational residuals must be minimised. After carrying out a least squares adjustment we know that we have the best possible solution based on the available observations.

Having determined a solution, it is important for the surveyor to be able to assess the quality of this solution. It is therefore necessary to somehow quantify the quality. By doing so, it can be verified whether the network meets the requirements. For example, a surveyor working, as a contractor will know before delivery whether or not and to what extent the quality of the network meets the client's demands. This works two ways:

- a network of poor quality will not be accepted by the client, this will obviously cause problems;
- a network with a far better quality than asked for by the client, is often undesirable from a cost-efficiency point of view.

The quality of a network, whether already measured or only existing as a design, can be assessed in terms of precision and reliability. Both precision and reliability can be quantified by parameters, which will be discussed in the following chapters.

By designing a network keeping in mind the demands on precision and reliability, it is possible to control the quality. However, designing a 'perfect' network is not enough. Practice has learned that, mainly due to human failures, about 1 in every 100 observations is erroneous. This means that the quality control will have to include some sort of statistical testing, in order to clear the result of possible outliers. The effectiveness of the testing will depend on the reliability of the network. The more reliable a network is the higher the probability that outliers will be detected by the testing.

The explanation above should clarify the relationship between the least squares adjustment, the precision and reliability concept, and the statistical testing. To summarise, one could say that:

- the least squares adjustment will produce the best possible result, given the available data;
- the statistical testing checks the result in order to make it 'error-free';
- the precision and reliability parameters quantify the quality of the result.


### 4.1.2. Quality Control in Network Planning

As explained in the preceding paragraph, the quality of a network can be assessed in terms of precision and reliability. This valuation may take place before the start of the actual measurements in the field, namely during the planning or design of the network. Usually the study of topographic maps of the area and reconnaissance in the field precedes the initial design. The outcome of the (initial) network design depends on the purpose of the network and on related demands on precision and reliability. Furthermore the location of the known stations and the characteristics of the area (e.g. mountainous or flat) can affect the design.

A number of general rules of thumb apply for network design:

- Aim for a balanced distribution of known stations over the network. Moreover, the integration of known stations should be acceptable from a precision and reliability point of view, e.g. an intersection by at least three, preferably four directions.
- Try to include loops in the network, keeping in mind that the lesser the number of stations in a loop, the better the reliability. In 2D networks the reliability of loops can be improved by measuring directions and distances to temporarily monumented auxiliary stations.
- Strive for network sides of approximately equal length.

Precision and reliability of a network design can be examined, for both free and constrained network, based on the following input:

- Approximate coordinates of all stations, e.g. scaled off from topographic maps. (Since observation readings are not available, approximate coordinates can not be computed.)
- Observations, i.e. not the actual readings but station and target, and the intended type of observation.
- Base stations (in case of a free network design).
- Known stations (in case of a constrained network design).
- The stochastic model of observations and known stations, i.e. a-priori standard deviations.

The output of the design computation is:

- Absolute and relative standard ellipses.
- Criterion circles (in case of a free network design).
- A-posteriori standard deviations of observations.
- A-posteriori standard deviations of stations.
- Minimal Detectable Bias (MDB) of observations.
- Normalised Minimal Detectable Bias (MDBn) of observations.
- Minimal Detectable Bias (MDB) of known stations (in case of a constrained network design).
- Bias to Noise Ratio (BNR) of observations.
- Bias To Noise Ratio (BNR) of known stations (in case of a constrained network design).

Based on this output, the network can be further improved until the requirements are satisfied. The design process is presented by the scheme in Figure 4.1.1.


Figure 4.1.1: Design process

### 4.2. Least Squares Adjustment

In this chapter the basic ideas of least squares adjustment are introduced, together with the concepts of mathematical and stochastic models. Furthermore, important notions used in MOVE3 such as approximate values, nuisance parameters, constraints and adjustment in phases, are reviewed.

### 4.2.1. General

It is common practice for a surveyor to carry out extra measurements when establishing a geodetic network. This will enable compensation for a possible loss of observations and, more important, the quality of the network can be improved. As a consequence of the extra measurements there is no longer a unique solution, which satisfies the conditions in the network exactly (e.g. the angles of a triangle should add up to 200 gon). Therefore a method is needed to correct the observations in order to make them meet the conditions. The amount by which each observation has to be corrected is called the observation residual. The least squares adjustment method will make the observations fit into the model by minimising the sum of squares of the observation residuals. The residuals are referred to as least squares corrections.

Any least squares adjustment model consists of two equally important components: the mathematical model and the stochastic model. The mathematical model is a set of relations between the observations and the unknowns. The stochastic model describes the expected error distribution of the observations.

### 4.2.2. Mathematical Model

Usually in a survey the observables themselves are not the quantities which we are aiming for. Instead, we use the observations to determine unknown parameters, e.g. the coordinates of stations in a network. The observations are expressed as a function of the parameters in the so-called functional or mathematical model.

In some cases the model representing the relations between the observables and the unknown parameters is very simple. The relation, for instance, in a 1dimensional levelling problem between the observed height differences and the unknown heights is simply linear:

$$
\Delta h_{i j}=h_{j}-h_{i}
$$

More complicated is the case for a GNSS/GPS network where the unknowns are coordinates ( $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ ) to be determined in a reference system different from that of the observed baselines $\Delta X$ :

$$
\Delta \mathrm{X}_{\mathrm{ij}}=\text { function }\left(\alpha, \beta, \gamma, \mu, \mathrm{X}_{\mathrm{i}}, \mathrm{Y}_{\mathrm{i}}, \mathrm{Z}_{\mathrm{i}}, \mathrm{X}_{\mathrm{j}}, \mathrm{Y}_{\mathrm{j}}, \mathrm{Z}_{\mathrm{i}}\right)
$$

with

$$
\alpha, \beta, \gamma, \mu \text { as transformation parameters. }
$$

As the least squares approach requires linear equations, the model above must be linearised. Usually this means that a number of iterations are needed to reach a solution. Moreover, approximate values for the coordinate unknowns in the adjustment are required. Bad approximate values can lead to an increasing number of iterations or, in the worst case, to no convergence at all.

Since coordinates are our main concern, other unknowns in the mathematical model are not always useful for us. Unknowns such as the transformation parameters mentioned above, are called additional or nuisance parameters. Typical nuisance parameters are transformation parameters, scale factors, azimuth offsets, orientation unknowns and refraction coefficients. Some of these parameters can be kept fixed at a certain value, in which case they are not corrected in the adjustment. Whether or not to fix parameters is a question that cannot easily be answered. We must always be careful to avoid overconstraining as well as overparameterisation. The introduction of refraction coefficients for example, could result in the absorption of systematic effects, which are not caused by refraction. However, ignoring the refraction, when in fact it does have an influence on the measurements, will cause an equally unfavourable effect. The success of what could be referred to as 'tuning' of the model will depend largely on the user's expertise.

Scale factors are especially important in 'tuning' the model. In MOVE3 up to 10 scale factors are allowed per network, but in most cases one scale factor is sufficient. The aim of introducing a free scale factor in the adjustment is to overcome a possible bias in the internal scale of the measurement equipment and, in more general terms, to prevent the overconstraining of the network in a free network adjustment. A free scale factor will 'shrink' or 'blow up' the network in order to make it fit onto the known stations in constrained adjustments. As a result, in some situations a free scale factor may obstruct the statistical testing of known stations. An outlier in the coordinates of a known station could remain undetected when, due to the 'shrink' or 'blow up' effect, the network can still be forced to fit the known stations without any rejections. The outlier in question is absorbed by the scale factor, which will consequently have a value distinctly different from 1.0. It is therefore recommended to examine the value of the scale factor after the adjustment and, in case of doubt, to rerun the adjustment with a fixed scale.

An adjustment will fail when the mathematical model, as represented by the design matrix and normal matrix (see paragraph 4.2.5, Formulae), is singular. Singularity is caused by:

- an ill-posed problem;
- an improperly formulated model.

The problem may be ill-posed because we expect too much from the observations, or because too few observations were included. An example of an ill-posed problem is the determination of the 2D coordinates of an unknown station by a single horizontal direction from another station. A model is improperly formulated, when too many parameters are included. In general an improperly formulated model does not correctly represent the existing physical reality.

For computerised solutions to least squares problems the ill-conditioning of the normal matrix N could result in singularity. Singularity is the limiting case of illconditioning. An ill-conditioned matrix can become 'singular' as a result of the internal accuracy limits of the computer hardware. An example of an ill-conditioned problem is the intersection of a station by two or more nearly parallel directions.

Apart from the problems mentioned above, an adjustment cannot be solved unless the location, orientation and scale of the network are established, i.e. a 'datum' must be defined. This is done by imposing constraints on the solution. The minimum number of constraints depends on the dimension of the network:

- In case of a 3D network there are 3 translations, 3 rotations and 1 scale factor. The singularity is then eliminated by fixing at least 7 coordinates of 3 stations (X East ${ }_{1}, \mathrm{Y}$ North $_{1}, \mathrm{~h}_{1}, \mathrm{X}$ East ${ }_{2}, \mathrm{Y}$ North $_{2}, \mathrm{~h}_{2}, \mathrm{~h}_{3}$ ).
- In case of a 2D network there are 2 translations, 1 rotation and 1 scale factor. The singularity is then phased out by fixing at least 2 base stations ( X East ${ }_{1}$, Y North ${ }_{1}, X$ East $_{2}, \mathrm{Y}$ North $_{2}$ ).
- In case of a 1D network the location of the network is determined by keeping one height fixed.


### 4.2.3. Stochastic Model

A geodetic observable, such as a direction, distance or height difference, is a random or stochastic variable. A stochastic variable cannot be described by a single and exact value because there is an amount of uncertainty involved in the measurement process. For example, repeatedly measuring the distance between two stations will result in a range of different values. This variation is accounted for by a probability distribution. This means that in addition to the mathematical model, it is necessary to formulate a second model which describes the stochastic deviations of the observations; the stochastic model.

For geodetic observables a normal probability distribution is assumed (figure 4.2.1). This distribution is based on the mean $\mu$ and the standard deviation $\sigma$.

figure 4.2.1: Normal Distribution
The mean $\mu$ represents the value of the mathematical expectation of the observable. The standard deviation is a measure of the dispersion or spread of the probability. The standard deviation characterises the precision of the observable(see paragraph 4.3.2, Precision). The square of $\sigma$ is called the variance. By definition there is a 0.684 probability that normally distributed stochastic variables will fall within a window limited by $-\sigma$ and $+\sigma$. For a window limited by $-2 \sigma$ and $+2 \sigma$ this probability is 0.954 . In general, the probability that a stochastic variable takes a value between $x_{1}$ and $x_{2}$ is equal to the area enclosed by the curve, and the $x_{1}$ and $x_{2}$ ordinates. This is the shaded area in figure 4.2.1.

It is possible for two or more observables to be interdependent or correlated. This means that a deviation in one observable will influence the other. The correlation between two observables $x$ and $y$ is mathematically expressed by the covariance $\sigma_{x y}$ . The covariance is also used in the correlation coefficient, defined as:

$$
\rho=\frac{\sigma_{x y}}{\sigma_{x} \sigma_{y}}
$$

The coefficient takes values between minus and plus one:

$$
-1 \leq \rho \leq 1
$$

If the observables are not interdependent it follows that $\rho=0$.
The vector elements (DX, DY, DZ) of a GNSS/GPS baseline are an example of correlated observables. To express this correlation a $3 \times 3$ matrix is used. In MOVE3 this symmetric matrix is a combination of standard deviations and correlation coefficients:

$$
\left(\begin{array}{ccc}
\sigma_{D X} & & \\
\rho_{D X D Y} & \sigma_{D Y} & \\
\rho_{D X D Z} & \rho_{D Y D Z} & \sigma_{D Z}
\end{array}\right)
$$

or practical reasons, and because their influence in the adjustment is limited, correlations are sometimes disregarded.

In essence the stochastic model consists of a choice for the probability distribution of the observables. Practically this means that for each observable a standard deviation $\sigma$ is chosen. The value for $\sigma$ is based on knowledge about the measurement process (conditions in the field, type of instrument) and experience. The standard deviation of most observables is supposed to consist of an absolute part, and a relative part. In the relative part the dependence on the distance between station and target, which characterises the precision of most observables, is accounted for. The thus defined standard deviations are entered in the variancecovariance matrix $Q_{y}$ (see paragraph 4.2.5, Formulae). The precision of the unknowns in the adjustment depends on the precision of the observables given in $Q_{y}$, and on the propagation of this precision through the mathematical model.

### 4.2.4. Free and Constrained Adjustments

The adjustment of a network is usually subdivided into two separate steps or phases:

- free network adjustment;
- constrained adjustment.

This approach is intended to separate the testing of observations and known stations.

A free network can be defined as a network of which the geometrical layout is determined by the observations only. The position, scale and orientation of the network are fixed by a minimum number of constraints, through the base stations. Thus, the base stations impose no extra constraints on the adjustment solution. In a free network adjustment the emphasis is laid on the quality control of the observations, rather than on the computation of coordinates. Selecting other stations to fix the position, scale and orientation will change the coordinates, but not the statistical testing as implemented in MOVE3.

Having eliminated possible outliers in the observations in the free network adjustment, the network can be connected to the known stations. This does impose extra constraints on the solution. Now the emphasis is on the analysis of the known stations and on the computation of the final coordinates. There are three types of constrained adjustments:

- In a pseudo least squares constrained adjustment the coordinates of the known stations are kept at their original value, i.e. they do not receive a least squares correction. Also the standard deviations of the known stations do not change in the adjustment.
- In a weighted constrained adjustment however, the known stations do receive a correction. The known stations are considered as observations, with their respective standard deviations.
- In an absolute constrained adjustment the coordinates of the known stations are kept at their original value, i.e. they do not receive a least squares correction. The standard deviations of the known stations will be kept 0 during the adjustment. The known stations are considered as observations, with standard deviations 0 .

The choice of the constrained adjustment type does not influence the testing of the known stations. For testing the known stations in all cases the specified standard deviations will be used.

### 4.2.5. Formulae

The (linearised) mathematical model is:

$$
\underline{y}=A x+\underline{e}+a
$$

with

$$
\begin{aligned}
\underline{L} & =(m) \text { vector of observations; } \\
\underline{e} & =(\mathrm{m}) \text { vector of corrections; } \\
A & =(\mathrm{m} \times \mathrm{n}) \text { design matrix; } \\
x & =(\mathrm{n}) \text { vector of unknowns; } \\
a & =(\mathrm{m}) \text { vector of constants. }
\end{aligned}
$$

The stochastic model is:

$$
Q_{y}=\sigma^{2} Q=\frac{1}{\sigma^{2}} P^{-1}
$$

with
$Q_{y}=(\mathrm{mxm})$ variance-covariance matrix;
$\sigma^{2}=$ a-priori variance-of-unit-weight;
$Q \quad=(\mathrm{m} \times \mathrm{m})$ weight coefficient matrix;
$P=(\mathrm{m} \times \mathrm{m})$ weight matrix.

The least squares criterion is:

$$
\underline{e}^{t} P \underline{e}=\text { minimum }
$$

The solution is:

$$
\begin{aligned}
& \hat{\hat{x}}=\left(A^{t} P A\right)^{-1} A^{t} P(\underline{y}-a) \\
& s^{2}=\frac{e^{t} P e}{m-n}
\end{aligned}
$$

with
( $\left.A^{t} P A\right)=\mathrm{N}=(\mathrm{n} \times \mathrm{n})$ normal matrix;
$s^{2}=$ a-posteriori variance-of-unit-weight.

The variance-covariance matrix of the unknowns:

$$
Q_{\hat{x}}=\sigma^{2} N^{-1}
$$

The a-priori $\sigma^{2}$ is used, not the a-posteriori $\mathrm{s}^{2}$

In case of a linearised mathematical model, the solution for the vector of unknowns x is available after a series of iterative updates $\Delta \hat{x}$ of the approximate values $x^{0}$ :

$$
\underline{\hat{x}}=x^{0}+\Delta \underline{\hat{x}}
$$

After each iteration, the new solution is compared with the previous one. If the difference between the two solutions satisfies the iteration criterion, the iteration process is ended and the last solution is regarded as final.

### 4.3. Precision and Reliability

The result of an adjustment must be precise and reliable. It is not enough for an observation to be precise, meaning that repetition will lead to a high degree of closeness. Observations should also be reliable, i.e. close to the true value. The accuracy or, more generally, the quality of a network can therefore be described by two elements: precision and reliability. In this chapter, the parameters, which quantify precision and reliability, will be reviewed.

### 4.3.1. General

Before proceeding to a more in-depth treatment, it is important to realise that precision and reliability are two separate notions. A measurement process can be very precise, but is therefore not necessarily reliable. On the other hand, a reliable process is not always a precise one. Compare the relative frequencies $f_{A}$ and $f_{B}$ of two measurement processes A and B (figure 4.3.1 and figure 4.3.2). The precision of process $A$ is better than of process $B$; the degree of closeness of the observations in process $A$ is higher. But although process $A$ is more precise, it is not very reliable. A systematic error has resulted in a shift of the frequency distribution, away from the true value $\mu$. Process B is not very precise, but definitely more reliable, since its distribution is close to the true value $\mu$.

In this chapter the precision and reliability concept is introduced, together with the tools implemented in MOVE3 for the analysis of precision and reliability of a network.

figure 4.3.1: Relative frequency of measurement proces $A$.

figure 4.3.2: Relative frequency of measurement proces $B$.

### 4.3.2. Precision

In chapter 4.2, Least Squares Adjustment, the standard deviation of an observable is introduced. It is necessary to describe an observation by both the measured value, and the standard deviation. The standard deviation expresses the stochastic variabilities of the observation. The precision of a network can be defined as the influence of the stochastic variabilities of the network observations on the coordinates.

In MOVE3 the a-posteriori standard deviations of all observations and stations are presented. For presenting the precision of stations standard ellipses are often used. Standard ellipses can be regarded as the 2-dimensional equivalent of standard deviations. These ellipses are also known as confidence ellipses. There is a certain level of confidence, that a station can be found within the area enwrapped by its ellipse. For standard ellipses the level of confidence is 0.39 (to get a level of confidence of 0.95 the axes have to be multiplied by a factor 2.5). Absolute standard ellipses represent the propagation of random errors through the mathematical model into the coordinates. Relative standard ellipses represent the precision between station pairs. The shape of an ellipse is defined by the semi major axis $A$ and semi minor axis $B$. The orientation of an absolute standard ellipse is defined by the angle Phi between the semi major axis and the Y-North axis of the coordinate system. The orientation of a relative standard ellipse is defined by the angle Psi between the semi major axis and the connecting line between station and target.

As seen in chapter 4.2, Least Squares Adjustment, the location, orientation and scale of a free network can be established by constraining a number of stations, the base stations. A difficulty in the interpretation of standard ellipses is that they depend on the choice of base stations. In the free network adjustment the magnitude of the absolute standard ellipses systematically increases with the distance from the base stations. As a different choice of base stations will result in differently shaped ellipses, it is difficult to draw conclusions about the precision of the network considering the standard ellipses alone.

Therefore in the free network adjustment so-called criterion circles are constructed. Criterion circles determine a theoretical model, representing a homogeneous precision of the network, computed with a similar choice of base stations. It is obvious that the ratio of the semi major axis of the standard ellipse A and the radius of the criterion circle $R, A / R$, is of more importance than the individual values. This ratio, however, still depends to a certain extend on the chosen base stations. Strictly speaking, this also applies for the ratio $A / R$ belonging to relative standard ellipses. In most cases however, the latter quantity behaves practically independent of the base stations.

The criterion model, expressed by the criterion circles, is regulated by the parameters $\mathbf{C 0}$ and $\mathbf{C 1}$. The C0 parameter controls the absolute part of the precision. The C1 parameter, the relative part, is a coefficient controlling the influence of the mutual distance on the relative precision between two stations. Often used empirically established values are: $\mathrm{C} 0=0 \mathrm{~cm}^{2}, \mathrm{C} 1=1 \mathrm{~cm}^{2} / \mathrm{km}$.

### 4.3.3. Reliability

The reliability of a network can be described in terms of the sensitivity to the detection of outliers. Reliability can be subdivided into internal and external reliability:

- Internal reliability is expressed by the Minimal Detectable Bias (MDB). The MDB presents the size of the smallest possible observation error, still detectable by the statistical test (datasnooping) with a probability equal to the power $\gamma$ of the test. A large MDB indicates a weakly checked observation or coordinate. Thus, the larger the MDB the poorer the reliability. If an observation is not checked at all, no MDB can be computed and the observation is marked as a 'free observation'. To compare the MDB values of different observation types the normalised Minimal Detectable Bias (MDBn) can be used. By relating the minimal detectable bias to the deviation of the observation the unit can be eliminated. For uncorrelated observations the normalised minimal detectable bias is equal to the minimal detectable bias divided by the standard deviation of the observation.
- Internal reliability can also be expressed in terms of the Redundancy Number. The (local) Redundancy number presents the contribution of an observation to the total redundancy. The local redundancy number lies in the closed interval $0 \leq r_{i} \leq 1$ (in the MOVE3 output file it is presented in a percentage $0-100 \%$ ). The correction multiplied by the redundancy number presents the size of the observation error. A large redundancy number indicates a strongly checked observation. Thus, the smaller the redundancy number the poorer the reliability. The redundancy number equals zero if the observation is not checked at all ('free observation'). The sum of the local redundancy numbers equals the total redundancy.
- External reliability is expressed by the Bias to Noise Ratio (BNR).The external reliability is used as a measure to determine the influence of a possible error in the observations on the adjusted coordinates. The BNR of an observation reflects this influence, whereby the size of the observation error is defined equal to the MDB of that particular observation. The BNR is a dimensionless parameter combining the influence of a single observation on all coordinates. A practical interpretation can be given if we regard the BNR as an upperbound for the ratio between the influence $\nabla$ of the MDB of an observation on any coordinate $x$, and the a-posteriori standard deviation $\sigma$ of this coordinate:

$$
\left|\frac{\nabla_{X}}{\sigma_{X}}\right| \leq B N R
$$

In other words: the BNR can be interpreted as the ratio between reliability and precision. It is desirable that the BNR is homogeneous for the entire network.

- External reliability can also be expressed in terms of Reliability Boxes. The Reliability Box (computed for every station) is used as a measure to determine the influence of a possible error in the observations or known coordinates on the adjusted coordinates. The size of the box is computed for a possible error (size of the MDB) in the observation or known coordinate having the largest influence on the coordinate. The box represents the area the station will be in (probability of $80 \%$ ) if all observations and known coordinates of the network are accepted by the statistical tests.

An important quality of the MDB, Redundancy Number and BNR is that they are independent of the choice of base stations. The Reliability Box is base dependent.

To illustrate the behaviour of the reliability parameters, consider the example presented in table 4.3.1.

| Observation | Reading (m) | MDB <br> $(\mathbf{m})$ | Red | BNR | W-test |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Distance A-B | 1051.426 | 0.048 | 82 | 10.8 | 0.76 |
| Distance A-B $+\Delta$ | 1051.476 | 0.048 | 82 | 10.8 | $3.53^{\star \star}$ |

table 4.3.1: Example internal and external reliability.
In this example the distance $\mathrm{A}-\mathrm{B}$ is an observation in a network which has been adjusted and tested. As follows from the last column of table 4.3.1, the testing has accepted this observation (critical value W-test $=3.29$ ). The internal reliability is given by an MDB of 0.048 m . This means that an error of this size will probably be detected by the W-test (see paragraph 4.4.3, W-test). To verify this, an error $\Delta$ of 0.05 m is introduced in the distance A-B. The network is now adjusted and tested again, including this simulated error. Indeed as seen from the second value in the last column, the observation is now rejected by the W-test because the critical value is exceeded.

The external reliability is given by a BNR of 10.8. This means that the influence of the MDB of 0.048 m on any coordinate in the network is smaller than 10.8 times the a-posteriori standard deviation of the coordinate. For general-purpose networks it is desirable to have a homogeneous external reliability. The network in the example is considered as homogeneous, when the BNR values of all observations are on the same level. Alternatively the network is inhomogeneous when the BNR of e.g. distance A-B strongly exceeds the BNR of other observations in the network. In that case the network is unstable or inhomogeneous, i.e. the reliability depends mainly on the correctness of one single observation.

### 4.4. Statistical Testing

The aim of statistical testing is to check whether the mathematical and stochastic models (see paragraph 4.2, Least Squares Adjustment) are a correct representation of 'reality'. Furthermore, it is important to detect possible outliers (blunders) in the observations, which could ruin the achievable accuracy. This makes statistical testing essential for the process of quality control. The statistical testing presented here is carried out together with the least squares adjustment. It is based on the analysis of least squares residuals. The detection of blunders can also be carried out prior to the adjustment by e.g. a check on loop misclosures or on erroneous station numbering. These checks are regarded as part of the pre-processing and are not further discussed here.

Three types of statistical tests are implemented in MOVE3: the F-test, W-test and Ttest. In this chapter the testing procedures are explained, preceded by a general description of hypothesis testing. In the final paragraphs attention is paid to the interpretation of the testing results and estimated errors.

### 4.4.1. General

The mathematical and stochastic models are based on a set of assumptions. This set is called a statistical hypothesis. Different assumptions will result in different hypotheses. Statistical testing is used to verify the hypotheses. A special set of assumptions is referred to as the null-hypothesis $\mathrm{H}_{0}$. This hypothesis implies that:

- there are no gross errors (blunders) present in the observations;
- the mathematical model gives a correct description of the relations between the observations and the unknown parameters;
- the chosen stochastic model for the observations appropriately describes the stochastic properties of the observations.

It is clear that there are two possible outcomes for the testing of a hypothesis: acceptance or rejection. A specific cut-off point or critical value decides over acceptance and rejection. The critical values establish a window of acceptance. The further beyond this window, the less certain the set of assumptions is satisfied. Critical values are determined by the choice of a level of significance $\alpha$. The probability that the critical value is exceeded, although the set of assumptions is valid, is equal to $\alpha$. In other words, $\alpha$ is the probability of an incorrect rejection. Alternatively the complementary level of confidence $1-\alpha$, is a measure of the confidence one can have in the decision.

In the testing of the null-hypothesis $\mathrm{H}_{0}$ there are two unfavourable situations that might occur:

- Rejection of $\mathrm{H}_{0}$ while in fact it is true. The probability of this situation occurring is equal to the significance level $\alpha$. This situation is called a Type I error (table 4.4.1).
- Acceptance of $\mathrm{H}_{0}$ while in fact it is false. The probability of this situation occurring is $1-\gamma$, with $\gamma$ the power of the test. This situation is called a Type II error (table 4.4.1).

| SITUATION | DECISION: accept $\mathrm{H}_{0}$ | DECISION: reject $\mathrm{H}_{\mathbf{0}}$ |
| :--- | :--- | :--- |
| $\mathrm{H}_{0}$ true | correct decision: <br> probability $=1-\alpha$ | Type I error: <br> probability $=\alpha$ |
| $\mathrm{H}_{0}$ false | Type II error: <br> probability $=1-\gamma$ | correct decision: <br> probability $=\gamma$ |

table 4.4.1: Testing of null-hypotheses.
In the following three paragraphs, methods are reviewed for testing the nullhypothesis and alternative hypotheses.

### 4.4.2. F-test

The F-test is a very commonly used multi-dimensional test for checking the nullhypothesis $\mathrm{H}_{0}$. The F-test is often called the overall model test, because it tests the model in general.

The F -value is given by the expression:

$$
F=s^{2} / \sigma^{2}
$$

with
$s^{2}=$ a-posteriori variance factor, depending on the computed residuals and the redundancy;
$\sigma^{2}=$ a-priori variance factor.

The F-value is tested against a critical value of the F-distribution, which is a function of the redundancy and the significance level $\alpha$. There are three sources of rejection, further described below, namely: gross errors, incorrect mathematical model, and incorrect stochastic model.

The information provided by the F-test, namely acceptance or rejection of the nullhypothesis is not very specific. Therefore if $\mathrm{H}_{0}$ is rejected, it is necessary to find the cause of the rejection by tracing errors in observations or assumptions. If we suspect that the $\mathrm{H}_{0}$ is rejected due to a gross error present in one of the observations, the W-test is required. The so-called datasnooping utilises the W-test in order to seek for errors in individual observations. The F- and W-test are linked by a common value of the power $\gamma$. This is the so-called $\mathbf{B}$-method of testing. The Wtest and the B-method are described in the next paragraph.

The $\mathrm{H}_{0}$ is likewise rejected when the mathematical model is incorrect, or not refined enough. For example: the vertical refraction coefficient is disregarded, or observations related to different datums are combined without solving the parameters of the datum transformation. In this case the mathematical model has to be improved, in order to prevent an inferior outcome.

Another source of rejection is a too optimistic a-priori variance-covariance matrix. Such a rejection can easily be remedied by increasing the input standard deviations of the observations. Of course we should remain aware of the purpose of statistical testing; the purpose is not to have all observations accepted, but rather to detect outliers and model errors.

Of course a combination of the three sources of rejection above can also occur.

### 4.4.3. W-test

A rejection of the F-test does not directly lead to the source of the rejection itself. In case the null-hypothesis is rejected, other hypotheses must be formulated which describe a possible error, or a combination of errors.

There are an infinite number of hypotheses, which can be formulated as an alternative for the null-hypothesis. The more complex these hypotheses become, the more difficult they will be to interpret. A simple but effective hypothesis is the socalled conventional alternative hypothesis, based on the assumption that there is an outlier present in one single observation while all others are correct. The onedimensional test associated with this hypothesis is the W-test.

The assumption of a single outlier is often very realistic. A strong rejection of the Ftest can often be traced back to a gross error or blunder in just one observation. There is a conventional alternative hypothesis for each observation, which implies that each individual observation is tested. The process of testing each observation in the network by a W -test is called datasnooping.

The size of the least squares correction alone is not always a very precise indicator when checking the observations for outliers. A better test quantity, though only suited for uncorrelated observations, is the least squares correction, divided by its standard deviation. For correlated observations, e.g. the three elements of a baseline, the complete weight matrix of the observations must be taken into account. This condition is fulfilled by the test quantity $W$ of the $W$-test, which has a standard normal distribution and is most sensitive for an error in one of the observations.

The critical value $W_{\text {crit }}$ depends on the choice of the significance level $\alpha_{0}$. If $W>W_{\text {crit }}$ (the W-test is rejected), there is a probability of $1-\alpha_{0}$ that the corresponding observation indeed holds an outlier. On the other hand there is a probability $\alpha_{0}$ that the observation does not hold an outlier, which means the rejection is unjustified. In geodesy values for $\alpha_{0}$ between 0.001 and 0.05 are most commonly chosen. The table 4.4.2 presents an overview of the $\alpha_{0}$-values and the corresponding critical values. The actual choice depends on how strict and rigid we want to test the observations. A very strict testing (a small critical value), will lead to a larger $\alpha_{0}$ and consequently an increasing probability of rejection of valid observations. A $\alpha_{0}=$ 0.001 means one false rejection in every 1000 observations. This has proven to be a workable choice in practice.

| sign. Level $\alpha_{0}$ | $\mathbf{0 . 0 0 1}$ | $\mathbf{0 . 0 1 0}$ | $\mathbf{0 . 0 5 0}$ |
| :--- | :--- | :--- | :--- |
| critical value W-test | 3.29 | 2.58 | 1.96 |

table 4.4.2: Significance level/critical value overview.
Essential for the B-method of testing is that both the F-test and the W-test detect an outlier with the same probability. For this purpose the power $\gamma$ of both tests is fixed on a level of usually 0.80 . The level of significance $\alpha_{0}$ of the W -test is also fixed, which leaves the level of significance $\alpha$ of the F-test to be determined. Having $\alpha_{0}$ and $\gamma$ fixed, $\alpha$ depends strongly on the redundancy in the network. For large-scale networks with many observations and a considerable amount of redundancy, it is difficult for the F-test to react on a single outlier. The F-test, being an overall model test, is not sensitive enough for this task. As a consequence of the link between the F-test and the W-test by which the power is forced at 0.80 , the level of significance $\alpha$ of the F-test will increase. Considering the above, it is common practice to always carry out the datasnooping, no matter the outcome of the F-test.

During the datasnooping, each individual observation is tested through a conventional alternative hypothesis. However, as mentioned before, other alternative hypotheses can be formulated as well. In MOVE3 a special hypothesis is implemented, in order to trace antenna height errors in GNSS/GPS baselines. The hypothesis is based on the fact that the direction of an antenna height error will coincide with the direction of the local vertical. The antenna height W-test has proven to be a very efficient tool, for instance in tracing 10-cm antenna height reading errors.

### 4.4.4. T-test

As discussed in paragraph 4.4.3, W-test, the W-test is a 1 -dimensional test, which checks the conventional alternative hypotheses. These hypotheses assume that there is just one observation erroneous at the time. This so-called datasnooping works very well for single observations, e.g. directions, distances, zenith angles, azimuths and height differences. However, for some observations such as GNSS/GPS baselines, it is not enough to test the DX-, DY-, DZ-elements of the vector separately. It is imperative to test the baseline as a whole as well.

For this purpose the T-test is introduced. Depending on the dimension of the quantity to be tested, the T -test is a 3 - or 2 -dimensional test. As with the W -test, the T-test is also linked to the F-test by the B-method of testing. The T-test has the same power as both other tests, but has its own level of significance and its own critical value (table 4.4.3 and table 4.4.4).

| sign. Level $\alpha_{0}$ | $\mathbf{0 . 0 0 1}$ | $\mathbf{0 . 0 1 0}$ | $\mathbf{0 . 0 5 0}$ |
| :--- | :--- | :--- | :--- |
| sign. level $\alpha$ (2-dim) | 0.003 | 0.022 | 0.089 |
| critical value T-test | 5.91 | 3.81 | 2.42 |

table 4.4.3: Overview of significance level/critical value for 2-dimensional T-test, based on $\alpha_{0}$ of W-test.

| Sign. Level $\alpha_{0}$ | $\mathbf{0 . 0 0 1}$ | $\mathbf{0 . 0 1 0}$ | $\mathbf{0 . 0 5 0}$ |
| :--- | :--- | :--- | :--- |
| sign. Level $\alpha$ (3-dim) | 0.005 | 0.037 | 0.129 |
| critical value T-test | 4.24 | 2.83 | 1.89 |

table 4.4.4: Overview of significance level/critical value for 3-dimensional T-test, based on $\alpha_{0}$ of W-test.
The T-test is equally useful when testing known stations. The datasnooping will test for an outlier due to, e.g. a typing error, in either the X East, or Y North, or hcoordinate. The deformation of a station might not be detected by the datasnooping when the deformation-shifts decomposed in X East, Y North, and h-direction are relatively small. For the testing of a possible deformation influencing both the $X$ East, and $Y$ North, and h-coordinate a different alternative hypothesis is needed. The 3-dimensional T-test on the complete coordinate triplet is better equipped to trace the deformation, although it will not be able to trace the exact direction in which the station has moved.

## $\triangle$ <br> The situation in which the W-test is accepted, and the associated T-test of the observation is rejected, which is not unlikely in practice, does not imply a contradiction. It is simply a matter of testing different hypotheses.

### 4.4.5. Interpreting Testing Results

When dealing with testing results we always have to keep in mind that a certain amount of probability is involved in the process, and that there is no 'absolute truth'. Statistics in general should be used with discretion, i.e. in conjunction with common sense, practical experience and external independent evidence.

As discussed in paragraph 4.4.2, F-test, a rejection of the F-test, meaning a rejection of $\mathrm{H}_{0}$, could be due to:

- gross errors or blunders;
- an incorrect mathematical model;
- an incorrect stochastic model.

The fact that also a combination of these rejection sources is possible makes it difficult to give any strict rules for drawing conclusions from the F-test value. In general a rejected F-test in combination with a pattern of rejected W-tests points to a model error. An incidental W-test rejection points to one or more gross errors or blunders.

Since the F-test, W-test and T-test are linked, it is best to interpret these tests combined:

- A rejected F-test in combination with a limited number of W-test (T-test) rejections usually points to one or more gross errors.
- If the F-test is rejected and all observations of a specific type (e.g. all zenith angles) are rejected as well, the problem could be the mathematical model, which needs correction or refinement. For instance, if all W-tests for the zenith angles are rejected, it may be useful to include refraction coefficients.
- If the F-test is rejected as well as most of the W-test values (without extremes), the problem could be the stochastic model. The input standard deviations are then too optimistic. On the other hand, if the F-test value is well below the critical value, and the W-test (T-test) values are all close to zero, the input standard deviations could be too pessimistic.

Suppose that the datasnooping on the observations in a network has resulted in a (limited) number of rejections. It is assumed that the rejections are not caused by mathematical model errors, and that obvious errors such as typing mistakes have been fixed. This leaves a number of options:

- Remove the corresponding observation.

This is a valid but rather abrupt way of handling rejections. Remember that the removal of observations decreases the redundancy, and therefore influences precision and reliability.

- Remeasure the corresponding observation.

Remeasuring observations is an obvious but often expensive way to eliminate rejections, especially when the fieldwork is already completed. It is therefore recommended to process the data on the site.

- Increase the standard deviation of the corresponding observations.

Increasing the standard deviation of an observation will always work, meaning that it will always result in lowering the F-, W-, T-test values. However, one should keep in mind that the goal is not to get all tests accepted, but rather to detect blunders or model errors.

- Ignore the rejections.

This option is obviously very risky and is only applicable in case the W-test values just exceed the critical value. It is useful to look at the estimate of the error involved with the rejection, and to see whether it is acceptable or not. We also have to remember that, depending on the level of significance, there is always a probability of rejecting a valid observation.

It must be stressed that an observation must never be edited in order to make it better fit in with the other observations of the network, unless there is clear evidence of the source of the errors, e.g. a typing error.

### 4.4.6. Estimated Errors

The size of the error responsible for the rejection of an observation or known coordinate is estimated by MOVE3. This so-called estimated error is a useful tool, but should be handled with care:

- Only the estimated error associated with the largest W-test or T-test should be considered.
- As far as the W-test is concerned, the estimated error is based on the conventional alternative hypothesis that just one observation or known coordinate contains an error. Consequently if more errors are present in the network the result of the estimation could be meaningless, unless errors have been made (geographically) far apart.
- As far as the T-test is concerned, the estimated error is based on the hypothesis that just one GNSS/GPS baseline or known station contains an error. Consequently if more errors are present in the network the result of the estimation could be meaningless, unless errors have been made (geographically) far apart.
- Known coordinates are tested by the W-test and T-test in a constrained network adjustment. The test results and estimated errors are only meaningful when observational errors have been filtered out in the foregoing free network adjustment and testing.


## 5. Lists

```
5.1. List of Map Projections and Constants
Local (proposed values):
\begin{tabular}{ll} 
Projection type: & Stereographic \\
Longitude of Origin: & \(0^{\circ}\) \\
Latitude of Origin: & \(0^{\circ}\) \\
Point Scale Factor: & 1.0 \\
False Easting: & 0 m \\
False Northing: & 0 m \\
Ellipsoid: & Bessel 1841
\end{tabular}
Universal Transverse Mercator (UTM):
Projection type: Transverse Mercator
\begin{tabular}{ll} 
Central Meridian: & \begin{tabular}{l} 
Depending on the zone starting at \(177^{\circ} \mathrm{W}\left(183^{\circ} \mathrm{E}\right)\) and \\
increasing with \(6^{\circ}\) eastward to \(177^{\circ} \mathrm{E}\). The zone numbering \\
in this case starts at 1 and increases to 60.
\end{tabular} \\
\begin{tabular}{ll} 
Latitude of Origin: & \(0^{\circ}\) (the equator)
\end{tabular} \\
Scale Factor: & 0.9996 \\
False Easting: & 500000.0 m \\
False Northing: & 0.0 m on the northern hemisphere \\
& 10000000.0 m on the southern hemisphere \\
Ellipsoid: & International (Hayford)
\end{tabular}
```


## Austria:

```
Projection type: Gauss-Krüger
Central Meridian: \(\quad 10^{\circ} 20^{\prime} 0.0^{\prime \prime} \mathrm{E}\left(28^{\circ}\right.\) east of Ferro)
\(13^{\circ} 20^{\prime} 0.0^{\prime \prime} \mathrm{E}\left(31^{\circ}\right.\) east of Ferro)
\(16^{\circ} 20^{\prime} 0.0^{\prime \prime} \mathrm{E}\left(34^{\circ}\right.\) east of Ferro)
(Ferro is \(17^{\circ} 40^{\prime} 0.0^{\prime \prime}\) west of Greenwich)
Latitude of Origin: \(\quad 0^{\circ}\) (the equator)
Scale Factor: \(\quad 1.0\)
False Easting: 0 m
\begin{tabular}{ll} 
False Northing: & 0 m \\
Ellipsoid: & Bessel 1841
\end{tabular}

\section*{Belgium:}
\begin{tabular}{ll} 
Projection type: & Lambert 72 \\
Longitude of Origin: & \(04^{\circ} 21^{\prime} 24.983^{\prime \prime} \mathrm{E}\) \\
Latitude of Origin: & \(50^{\circ} 47^{\prime} 57.704^{\prime \prime} \mathrm{N}\) \\
Standard Parallel 1: & \(49^{\circ} 50^{\prime} 00.000^{\prime \prime} \mathrm{N}\) \\
Standard Parallel 2: & \(51^{\circ} 10^{\prime} 00.000^{\prime \prime} \mathrm{N}\) \\
False Easting: & 150000.0 m \\
False Northing: & 5400000.0 m \\
Ellipsoid: & International (Hayford)
\end{tabular}

France:
Projection type: Lambert
Longitude of Origin: \(\quad 02^{\circ} 20^{\prime} 13.94952^{\prime \prime} \mathrm{E}\) ( 0 Grad Paris)
Latitude of Origin: standard parallel
Standard Parallel: \(\quad 49^{\circ} 30^{\prime} 00.0^{\prime \prime}\) E ( 55 Grad) Zone Nord
460 48' 00.0" E (52 Grad) Zone Centre
44ㅇ 06' 00.0" E (49 Grad) Zone Sud
42º 09' 54.0" E (46.85 Grad) Corse
Scale Factor: \(\quad 0.9998773411\) Zone Nord
0.9998774203 Zone Centre
0.9998774993 Zone Sud
0.9999447095 Corse

False Easting: \(\quad 600000.0 \mathrm{~m}\)
False Northing: \(\quad 200000.0\) m
Ellipsoid: Clarke 1880

\section*{Germany:}

Projection type: Gauss-Krüger
\begin{tabular}{|c|c|}
\hline \multirow[t]{5}{*}{Central Meridian:} & \(3^{\circ} \mathrm{E}\) for zone number 1 \\
\hline & \(6^{\circ} \mathrm{E}\) for zone number 2 \\
\hline & \(9^{\circ} \mathrm{E}\) for zone number 3 \\
\hline & \(12^{\circ} \mathrm{E}\) for zone number 4 \\
\hline & \(15^{\circ} \mathrm{E}\) for zone number 5 \\
\hline Latitude of Origin: & \(0^{\circ}\) (the equator) \\
\hline Scale Factor: & 1.0 \\
\hline \multirow[t]{5}{*}{False Easting:} & 1500000.0 m for zone number 1 \\
\hline & 2500000.0 m for zone number 2 \\
\hline & 3500000.0 m for zone number 3 \\
\hline & 4500000.0 m for zone number 4 \\
\hline & 5500000.0 m for zone number 5 \\
\hline False Northing: & 0 m \\
\hline Ellipsoid: & Bessel 1841 \\
\hline \multicolumn{2}{|l|}{Great Britain:} \\
\hline Projection type: & Transverse Mercator \\
\hline Central Meridian: & \(358^{\circ} \mathrm{E}\) \\
\hline Latitude of Origin: & \(49^{\circ} \mathrm{N}\) \\
\hline Scale Factor: & 0.9996013 \\
\hline False Easting: & 400000.0 m \\
\hline False Northing: & -100000.0 m \\
\hline Ellipsoid: & Airy \\
\hline
\end{tabular}

\section*{Maryland (as an example for the US):}

Projection type: Lambert
Longitude of Origin: \(\quad 283^{\circ} 00^{\prime} 0.0^{\prime \prime} \mathrm{E}\)
Latitude of Origin: \(\quad 37^{\circ} 40^{\prime} 0.0^{\prime \prime} \mathrm{N}\)
Standard Parallel 1: \(\quad 38^{\circ} 18^{\prime} 0.0^{\prime \prime} \mathrm{N}\)
Standard Parallel 2: \(\quad 39^{\circ} 27^{\prime} 0.0^{\prime \prime} \mathrm{N}\)
False Easting: \(\quad 400000.0 \mathrm{~m}\)
\begin{tabular}{ll} 
False Northing: & 0 m \\
Ellipsoid: & Clarke 1866
\end{tabular}

\section*{The Netherlands:}

Projection type: Stereographic (RD)
Longitude of Origin: \(\quad 05^{\circ} 23^{\prime} 15.500^{\prime \prime} \mathrm{E}\)
Latitude of Origin: \(\quad 52^{\circ} 9^{\prime} 22.178{ }^{\prime \prime} \mathrm{N}\)
Point Scale Factor: 0.9999079
False Easting: \(\quad 155000.0 \mathrm{~m}\)
False Northing: \(\quad 463000.0 \mathrm{~m}\)
Ellipsoid: Bessel 1841

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\subsection*{5.3. Glossary}

Azimuth: The angle between the North direction and the direction to a target.
Base stations: The base stations are the stations to which the network is connected in a free network adjustment. By fixing the minimum number of coordinates (i.e. without extra constraints) only location, orientation and scale of the network is defined.

Baseline: A three-dimensional vector in WGS'84 between two stations.
Bias to Noise Ratio (BNR): The BNR gives the influence of a possible outlier (size MDB) in an observation on all coordinates. The BNR is regarded as the ratio between reliability and precision.

Cartesian coordinate system: An orthonormal coordinate system, with the same scale on all axes, with coordinates taken as segments on the axes denoted by, in case of a three dimensional system, (X,Y,Z).

Co linearity: The geometrical relation that three points are located on a straight line.
Constrained adjustment: In a constrained adjustment the network is connected to all known stations. MOVE3 discriminates between three types of constrained adjustments. In a pseudo constrained adjustment the known coordinates do not get corrections, and the standard deviations of the known stations do not change. In a weighted constrained adjustment the known coordinates are considered as normal observations and they will get corrections in the adjustment. In an absolutely constrained adjustment the known coordinates do not get corrections and the standard deviations of the known stations will be 0 after the adjustment.

Correlation: There is correlation between two quantities when these quantities are statistically dependent, i.e. the quantities influence each other. Correlation is expressed by \(\rho\), the correlation coefficient.

Criterion circle: The criterion circles determine a theoretical model representing a homogeneous precision, with respect to the base stations. Criterion circles are used when judging the standard ellipses (which depend on the choice of the base stations) in a free network adjustment.
Critical value: The predetermined value, which decides over acceptance or rejection of a hypothesis, associated with a statistical test.

Datasnooping: Statistical testing method in which every observation is tested separately by the W-test.

Datum: A mathematical model, the ellipsoid, designed to best fit the earth's surface as a whole or over a certain region. It is defined by giving the ellipsoidal parameters (semi major axis, flattening) and the parameters to fix the position of the ellipsoid.

Degree of freedom: see redundancy.
Design: When designing a network the configuration, consisting of known and unknown stations and the observations between them, is established before realization in the field. The quality of the network design can be rated in terms of precision and reliability.

Distance: The length of the vector between two stations. The slope distance represents the directly measured distance from instrument to reflector, while the horizontal distance represents the distance after reduction.

Eccentricity (e): One of the parameters defining the geometry of an ellipsoid:
\[
e^{2}=f(2-f)=\left(a^{2}-b^{2}\right) / a^{2}
\]
with
\(f=\) flattening;
\(a=\) semi major axis;
\(b=\) semi minor axis.
(See also flattening f.)
Ellipsoid: A mathematical figure formed by revolving an ellipse about its semi minor axis \(b\). An ellipsoid is usually defined by its semi major axis \(a\) and by its flattening, \(f=(a-b) / a\).

Ellipsoid height (h): The vertical distance to a mark above the ellipsoid.
Expectation value ( \(\mu\) ): The expectation value of a stochastic variable is the weighted mean value over all possible outcomes of the measurement process.

External Reliability: The maximum influence of an error of the size of the MDB on the final coordinate.

F-test: Overall model test, which is used to check the null-hypothesis.
Flattening ( \(f\) ): One of the parameters defining the geometry of an ellipsoid:
\[
f=(a-b) / a
\]
with
\(a=\) semi major axis;
\(b=\) semi minor axis.
Free network adjustment: Adjustment in which the location, orientation and scale of the network are defined by imposing a minimum number of constraints (i.e. by fixing only the base stations). A free network adjustment is used to test the observations before connecting the network to all known stations.

Geometrical relations: Collection of observation types: angle, perpendicular (3 points or 2 lines), co linearity, distance between point and line, parallelism and chanaige and offset.
Geoid: The gravity equipotential surface, which best approximates the mean sea level over the whole earth. The geoid serves as vertical datum for orthometric heights.
Geoid height ( \(\mathbf{N}\) ): The difference between ellipsoidal height and orthometric height: \(\mathrm{N}=\mathrm{h}-\mathrm{H}\).

GNSS/GPS: Global Positioning System, a satellite-based system for determining three-dimensional relative and absolute positions.

Hypothesis: Set of assumptions on the behaviour of stochastic variables. A special hypothesis is the null-hypothesis \(\mathrm{H}_{0}\) that implies the correctness of the stochastic and mathematical model and the absence of gross errors in the observations. Statistical testing is used to verify hypotheses.
Known station: A known station is a station of which coordinates are given with a certain precision prior to the adjustment. A network is connected to the known stations in a constrained adjustment.

Least squares adjustment: A method for obtaining unknowns from redundant observations based on the least squares criterion. The unknowns are related to the observations by the mathematical model. The least squares criterion says that the sum of squares of the corrections to the observations must be minimal. The least squares corrections are also known as residuals.

Map projection: Set of mathematical formulae, which transfers ellipsoidal coordinates (latitude and longitude) into plane coordinates (X East, Y North), and vice versa. The transformation results in distortion of geometric elements, distance and angle. The most commonly used map projections in geodesy are Transverse Mercator, Lambert and Stereographic projection.

Mathematical model: A model, which expresses the relationship between the observations and the unknowns in an adjustment.

Minimal Detectable Bias (MDB): The smallest outlier in an observation or a known coordinate, which can be detected by the W-test with a certain probability. This probability is equal to the power of the statistical testing.

Minimal Detectable Bias, normalised (MDBn): Describes the internal reliability of the network. For uncorrelated observations equal to the MDB divided by the standard deviation of the observation.

Observation: An observation is the realisation of a stochastic variable.
Orientation: The orientation is the reference direction of the theodolite. For each station where directions are observed, an orientation unknown is solved in the adjustment. Each extra measurement series results in an extra orientation unknown.

Orthometric height \(\mathbf{( H ) : ~ T h e ~ v e r t i c a l ~ d i s t a n c e ~ t o ~ a ~ m a r k ~ a b o v e ~ t h e ~ g e o i d . ~ V e r t i c a l ~}\) here means: along the direction of the local force of gravity.

Phase adjustment: An adjustment can be subdivided into phases; a free network adjustment and a constrained adjustment. The latter can be subdivided in a pseudo constrained adjustment, weighted constrained adjustment or an absolutely constrained adjustment.

Positioning: The determination of the coordinates of a station with respect to a welldefined coordinate system. By relative positioning coordinate differences are determined rather than absolute coordinates.

Precision: Precision represents the stochastic deviations in observations and coordinates. Precision is quantified by standard deviations and standard ellipses.

Probability: Probability is related to the frequency of occurrence of a specific outcome of the measurement process. Each outcome has an associated probability. The probability distribution describes the probabilities as related to the possible outcomes of a stochastic variable.

Pseudo least squares adjustment: Adjustment to connect the network to all known stations without corrections to the known coordinates.

Redundancy: The redundancy, also called the degree of freedom, in an adjustment is equal to the number of observations minus the number of unknowns.

Redundancy number: The (local) redundancy number presents the contribution of an observation to the total redundancy. The sum of the local redundancy numbers equals the degree of freedom.

Refraction coefficient: The influence of the refraction causes a curvature of light beams in vertical direction. This curvature can be modelled by a refraction coefficient \(k\). Under normal circumstances:
\[
k \approx 0.13
\]

The coefficient has a positive value if the beam is convex (curvature towards the earth), and a negative value if the beam is concave (curvature away from the earth).

Reliability: Reliability is the sensitivity of a network configuration to the detection of outliers. Reliability is quantified by Minimal Detectable Biases, Bias to Noise Ratios and External Reliability.
Singularity: Singularity occurs when one or more unknowns cannot be solved in the adjustment.

Standard deviation ( \(\sigma\) ): The standard deviation is the standardised measure of precision.

Standard ellipse: The precision of stations in a network is expressed by absolute standard ellipses. Relative standard ellipses represent the precision of coordinate differences between two stations. Standard ellipses are the 2D equivalent of standard deviations.

Stochastic model: Model, which describes the stochastic deviations of the observations and known coordinates in an adjustment.
Stochastic variable: A stochastic variable is a variable, which exhibits random or stochastic variabilities. Such a variable can not be described by a single and exact value, but is usually described by an expectation value and a standard deviation.

Terrestrial Observations: Terrestrial observations are observations obtained without involvement of satellites, e.g. directions, distances, zenith angles, azimuths, and height differences.

T-test: Statistical test used to trace an error in an observation or coordinate pair or triplet. Depending on whether a pair or triplet is tested, the test is two- or threedimensional.

Variance ( \(\sigma^{2}\) ): The variance is the square of the standard deviation.

Variance Component Analysis: The Variance Component Analysis provides a separate estimation of the a-posteriori variances for different observation types and for different groups of observations of the same type.

W-test (datasnooping): One-dimensional statistical test used to trace erroneous observations by testing so-called conventional alternative hypotheses. A conventional alternative hypothesis assumes an outlier in one single observation while the other observations are correct.

WGS'84: World Geodetic System 1984; the global reference system for GNSS/GPS observations since January 1987. In some cases its predecessor, WGS'72, is still utilised.

Zenith angle: The vertical angle between the direction to the zenith of the observer and the direction to a target. The zenith is the direction opposite to the local direction of the force of gravity.

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[^0]:    $\triangle$
    It is possible to solve for all 7-transformation parameters when observed GNSS/GPS coordinates are included. Observed GNSS/GPS coordinates are a specific observation type in MOVE3 (see paragraph 2.2.2, Observation Types).

